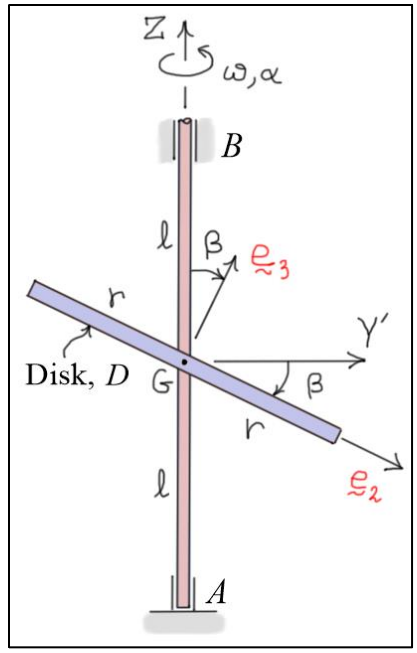
Intermediate Dynamics
Bearing Loads on a Shaft with a Misaligned Disk

<u>Example 15</u>: (Disk of mass *m* welded to light shaft)

Reference frames: (*R* is the fixed frame)

$S: \; \underset{\sim}{i}', \underset{\sim}{j}', \underset{\sim}{k}$ (rotates with the shaft; aligned with the shaft)

$D: \; \underset{\sim}{i}', \underset{\sim}{e}_2, \underset{\sim}{e}_3$ (rotates with the shaft; aligned with the disk)



Find:

$\underset{\sim}{A}, \underset{\sim}{B}$ … the bearing loads at *A* and *B*

T … the driving torque

Neglect weight forces.

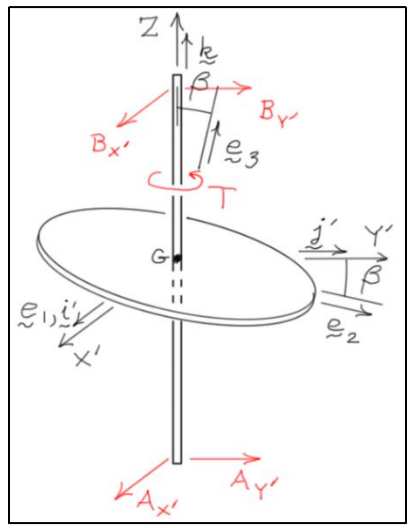
Solution:

Previous results:

$$\left[I_G\right]_D = m r^2 \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$${}^R\underset{\sim}{\omega}_D = \omega\,\underset{\sim}{k} = \omega\left(-S_\beta\,\underset{\sim}{e}_2 + C_\beta\,\underset{\sim}{e}_3\right)$$

$$\begin{aligned} \underset{\sim}{H}_G &= \tfrac{1}{4} m r^2 \omega\left(-S_\beta\,\underset{\sim}{e}_2 + 2C_\beta\,\underset{\sim}{e}_3\right) \\ &= \tfrac{1}{4} m r^2 \omega\left[\left(S_\beta C_\beta\right)\underset{\sim}{j}' + \left(C_\beta^2 + 1\right)\underset{\sim}{k}\right] \end{aligned}$$



The ***bearing loads*** at *A* and *B* can be found by applying the ***Newton/Euler equations of motion*** to the ***free-body diagram***.

Force Equations:

$$\sum \underset{\sim}{F} = \underset{\sim}{A} + \underset{\sim}{B} = m\,{}^R\underset{\sim}{a}_G = \underset{\sim}{0} \quad \Rightarrow \quad \begin{cases} A_{X'} + B_{X'} = 0 \\ A_{Y'} + B_{Y'} = 0 \end{cases} \quad \text{(scalar force equations)}$$

Moment Equations:

$$\sum \underset{\sim}{M}_A = \left(\underset{\sim}{r}_{B/A} \times \underset{\sim}{B}\right) + T\,\underset{\sim}{k} = \left(-2\ell B_{Y'}\right)\underset{\sim}{i}' + \left(2\ell B_{X'}\right)\underset{\sim}{j}' + T\,\underset{\sim}{k}$$

$$\sum \underset{\sim}{M}_A = \left(\underset{\approx}{I}_G \cdot {}^R\underset{\sim}{\alpha}_B\right) + \left({}^R\underset{\sim}{\omega}_B \times \underset{\sim}{H}_G\right) = \tfrac{1}{4} m r^2 \dot{\omega}\left[\left(S_\beta C_\beta\right)\underset{\sim}{j}' + \left(C_\beta^2 + 1\right)\underset{\sim}{k}\right] + \left(-\tfrac{1}{4} m r^2 \omega^2 S_\beta C_\beta\right)\underset{\sim}{i}'$$

Scalar moment equations:

$$\begin{aligned} -2\ell B_{Y'} &= -\tfrac{1}{4} m r^2 \omega^2 S_\beta C_\beta \\ 2\ell B_{X'} &= \tfrac{1}{4} m r^2 \dot{\omega} S_\beta C_\beta \\ T &= \tfrac{1}{4} m r^2 \dot{\omega}\left(C_\beta^2 + 1\right) \end{aligned}$$

… solving the equations gives $\Rightarrow$

$$\begin{aligned} B_{X'} &= \tfrac{1}{8\ell} m r^2 \dot{\omega} S_\beta C_\beta = -A_{X'} \\ B_{Y'} &= \tfrac{1}{8\ell} m r^2 \omega^2 S_\beta C_\beta = -A_{Y'} \end{aligned}$$