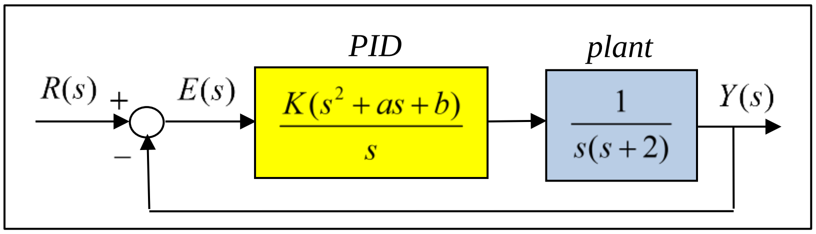
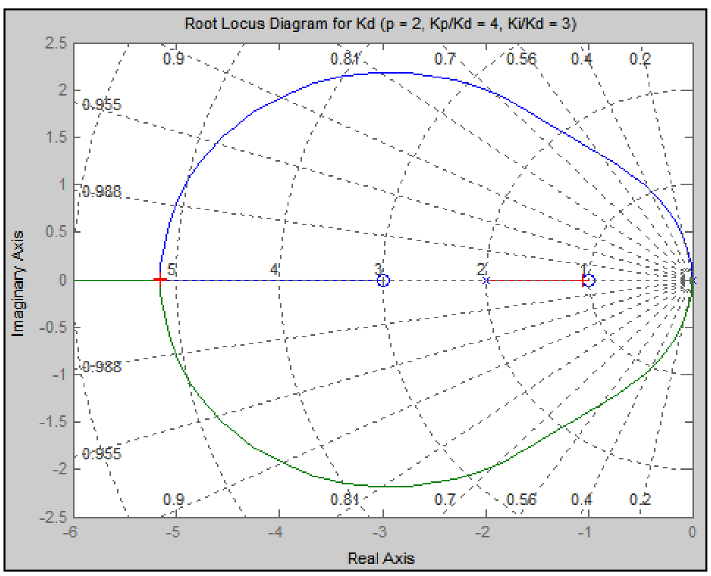
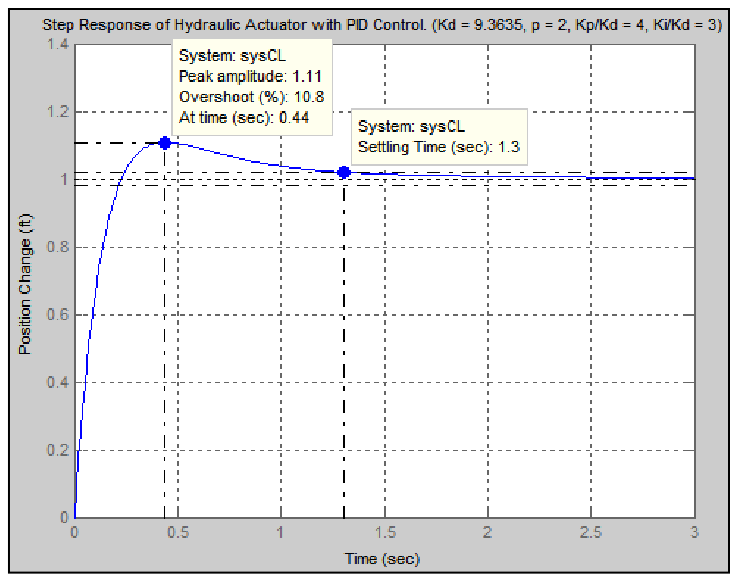
**Introductory Motion and Control**
**PID Root Locus Design with a First Order Noise Filter**

Previously we designed a ***PID compensator*** for a hydraulic actuator. Here, $K = K_D$ is the ***derivative gain***, $a = K_P / K_D$ is the ***ratio*** of the ***proportional*** to ***derivative*** gains, and $b = K_I / K_D$ is the ***ratio*** of the ***integral*** to ***derivative*** gains.

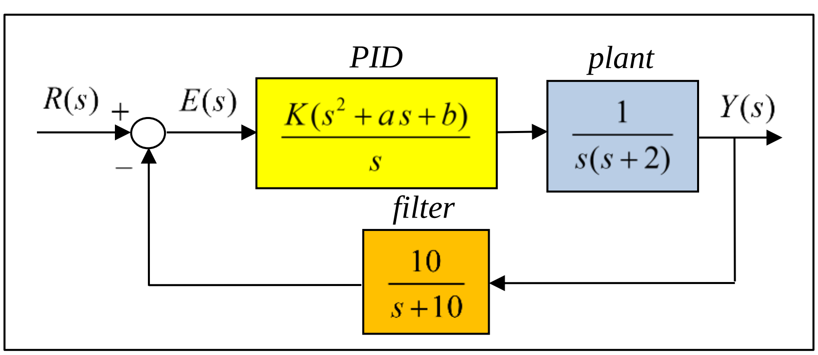



This is a ***type 2*** system and has ***zero steady-state error*** to ***both*** step and ramp inputs. The figures below show the ***root locus diagram*** for the parameter $K_D$ with $a = K_P / K_D = 4$ and $b = K_I / K_D = 3$, and a ***step response*** for $K_D = K \approx 9.36$.

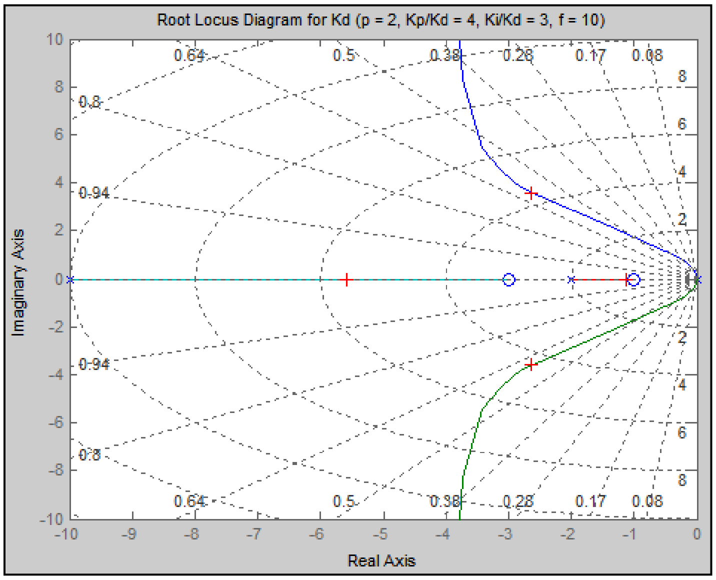


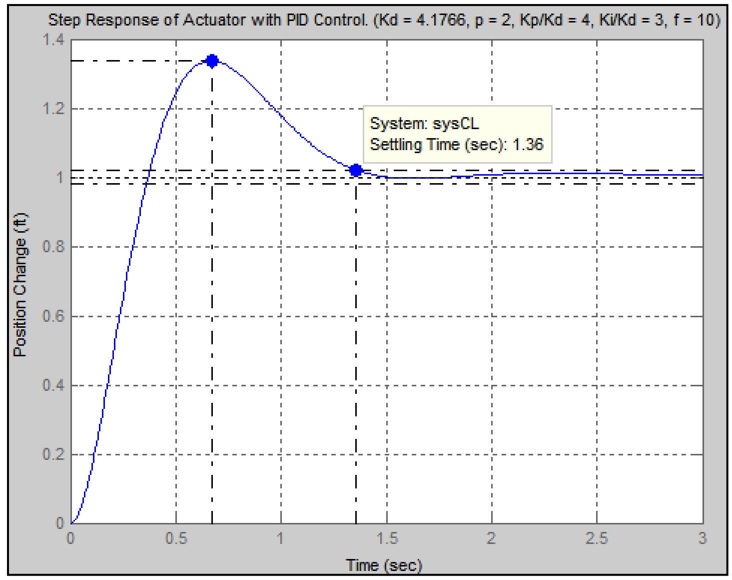




Now consider ***PID control*** of the same actuator with a normalized ***first-order noise filter*** in the feedback path as shown in the diagram below. The cut-off frequency of the filter is 10 (rad/sec).

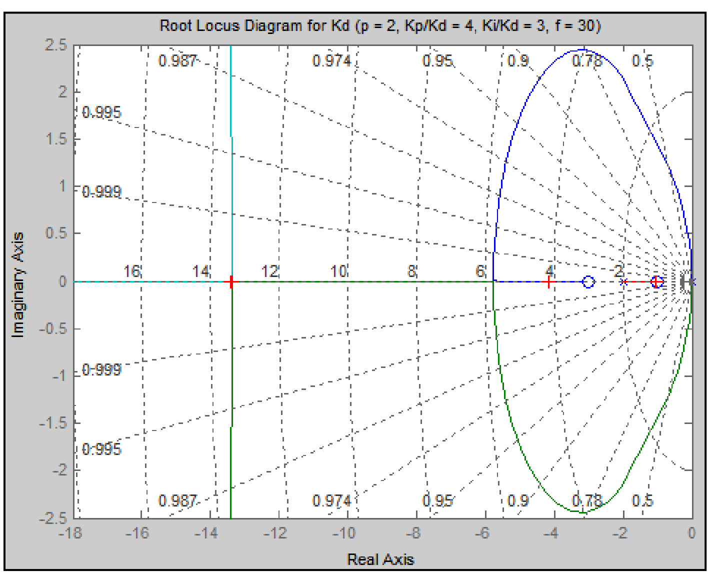


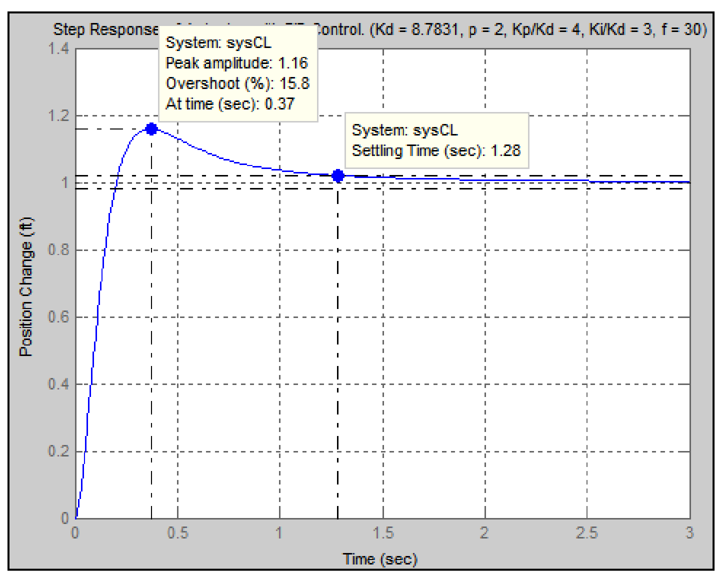
The figures below show the ***root locus diagram*** for the parameter $K = K_D$ with $a = K_P / K_D = 4$ and $b = K_I / K_D = 3$, and the ***step response*** for $K = K_D \approx 4.18$. The filter has had ***little effect*** on the ***settling time***, but it has ***increased*** the ***system overshoot***. Note on the ***root locus diagram*** the complex poles approach ***vertical asymptotes*** at $s = -4$ and ***do not*** break into the real axis as they did when no filter was present.




The figures below show the ***root locus diagram*** for the parameter $K = K_D$ with $a = K_P / K_D = 4$ and $b = K_I / K_D = 3$, and a ***step response*** for $K = K_D \approx 8.78$. Here, the filter cut-off frequency was changed from 10 to 30 (rad/sec). This causes the complex poles to ***break into*** the real axis ***before breaking out*** and moving along the vertical asymptotes. The step response is like the system with no filter but has a slightly ***higher overshoot***.



The figures below show the ***root locus diagram*** for the parameter $K = K_D$ with $a = K_P / K_D = 4$ and $b = K_I / K_D = 3$, and a ***step response*** for $K_D = K \approx 13.64$. Here, the cut-off frequency of the filter was ***increased*** further to 50 (rad/sec). As in the previous case, the complex poles break into the real axis before breaking out and moving along the asymptotes. This system has the ***same overshoot*** as the system with no filter, and it has a slightly ***lower settling time*** probably due to the higher gain.