

## Introductory Control Systems

### Dominant (or Insignificant) Poles

- The *slowest poles* of a system (those closest to the imaginary axis in the  $s$ -plane) give rise to the *longest lasting* terms in the transient response.
- If a pole or set of poles are *very slow compared to others* in the transfer function, they may *dominate* the transient response.
- If the transient response of the system is plotted *without accounting for* the transient response of the *fastest poles*, it may be found that there is *little difference* from the transient response of the original system. In this case, the fastest poles are called *insignificant*.

### Example: A Third Order System

Consider a third order system that has *one real and two complex conjugate poles*.

$$T(s) = \frac{K}{(s + p)(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

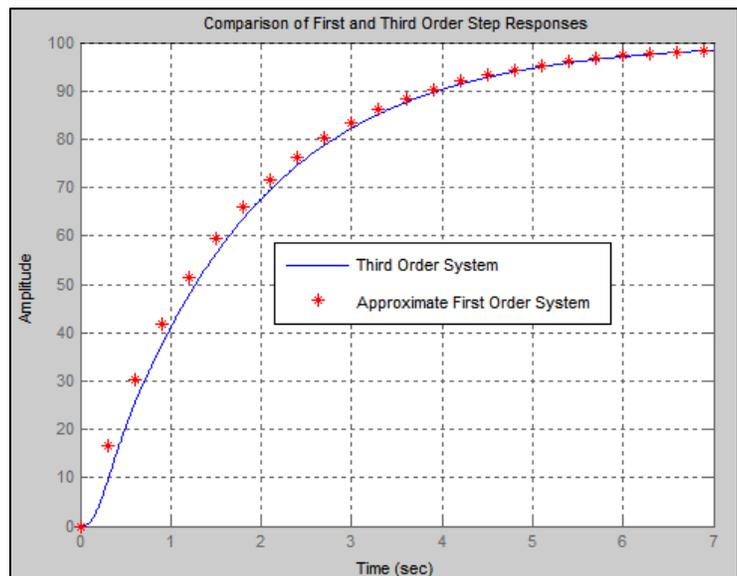
In general, all three poles contribute to the response of the system. However, some third-order systems exhibit dominant first-order or dominant second-order behavior.

### Dominant First Order Behavior

If  $\zeta\omega_n \geq 10p$ , the system exhibits *dominant first-order* behavior. The *approximate lower-order transfer function* is

$$T(s) \approx \frac{K/\omega_n^2}{(s + p)}$$

The plot on the right shows results using  $K = 6000$ ,  $\zeta = 0.6$ ,  $\omega_n = 10$ , and  $p = 0.6$ .



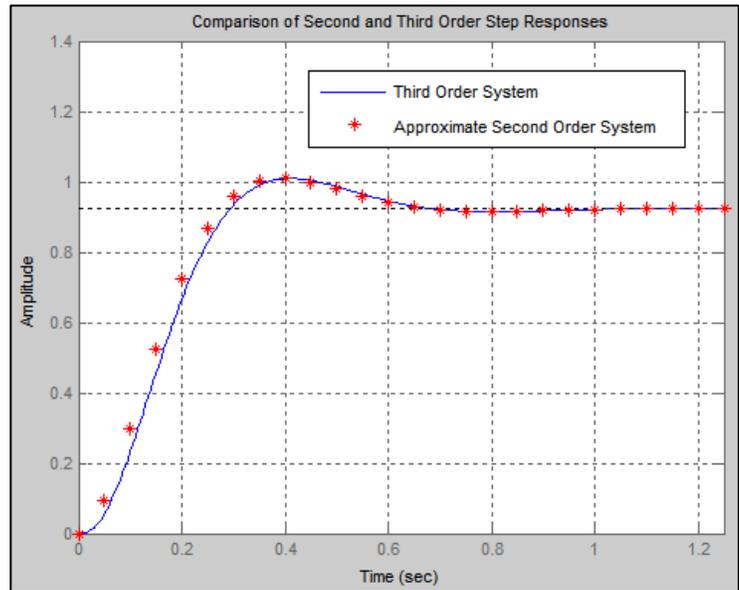
The response of the *first-order approximation* is *very close* to that of the *third-order system*. The *largest errors* occur *early* in the response. It is important when forming the *approximate transfer function* to *retain* the *steady-state parts* of the insignificant poles.

## Dominant Second Order Behavior

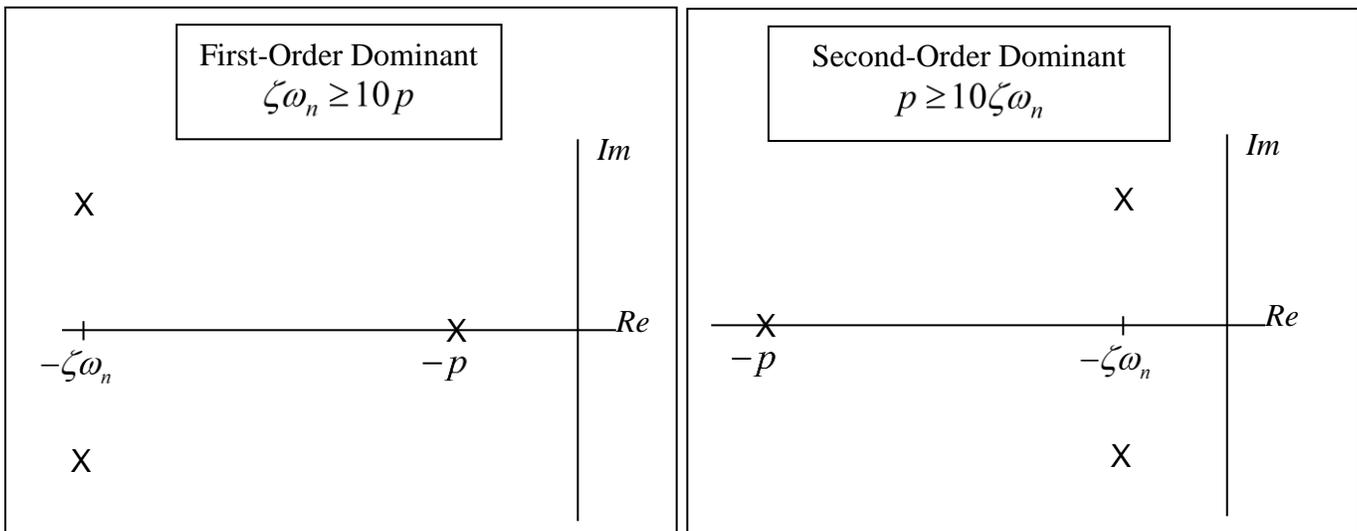
If  $p \geq 10\zeta\omega_n$ , the system exhibits *dominant second-order* behavior. The *approximate lower order transfer function* is

$$T(s) \approx \frac{K/p}{(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

The plot on the right shows results using  $K = 6000$ ,  $\zeta = 0.6$ ,  $\omega_n = 10$ , and  $p = 65$ .



In this case, the response of the *second-order approximation* is *very close* to that of the *third-order system*. As before, the *largest errors* occur *early* in the response.



### Notes:

- The concept of *pole dominance* can also be applied to *higher-order* systems. If a pole or set of poles are much closer to the imaginary axis than all other poles in the system, they may dominate the transient response. Having knowledge of the *physical origin* of these poles allows the analyst to focus on controlling the most important (dominant) parts of the system.
- The *simplest approach* for generating an approximate transfer function is to simply “drop” the *s-dependence* of the *insignificant* poles. The constant parts of these terms are kept, maintaining the correct steady-state response.