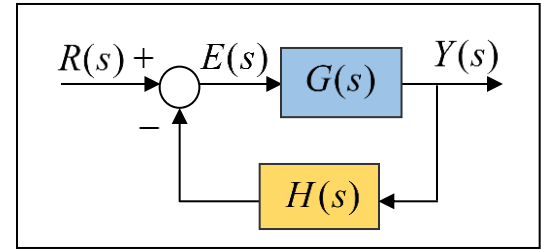


Introductory Control Systems

System Type and Steady-State Error

System Type

Consider the single loop feedback system with input $R(s)$ and output $Y(s)$ as shown at the right. The *system type* is determined by the form of the *loop* (or open loop) *transfer function* $GH(s)$. In general, the form of $GH(s)$ can be written as



Simple Closed Loop System

$$\boxed{GH(s) = \frac{N(s)}{s^n D(s)}} \quad (1)$$

where $N(s)$ and $D(s)$ represent polynomials in s , and s^n represents all powers of s that can be factored from the denominator. The number n determines the system type. It is said that the system is a “*type n*” system. As is shown below, the value of n determines the “*type*” of *steady-state error* the system will have for various input signals.

System Error Transfer Function and Steady-State Error

Using *block diagram reduction*, it can be shown that the *error transfer function* for the single loop system is

$$\boxed{\frac{E}{R}(s) = \frac{1}{1 + GH(s)}} \quad (2)$$

Combining equations (1) and (2) gives the general form

$$\boxed{\frac{E}{R}(s) = \frac{s^n D(s)}{(s^n D(s)) + N(s)}} \quad (3)$$

The *steady state error* is found using the *final value theorem*. Steady state errors for type “0” and type “1” systems for *step*, *ramp*, and *parabolic* inputs are calculated below. Given these results, the results for type “2” and higher systems are easily deduced.

Type “0” System

The steady-state error of a type “0” system to *step*, *ramp*, and *parabolic* inputs can be calculated as follows:

Step Input:

$$e_{ss} = \lim_{s \rightarrow 0} (s E(s)) = \lim_{s \rightarrow 0} \left(s \cdot \frac{E}{R} \cdot \frac{1}{s} \right) = \lim_{s \rightarrow 0} \left(\frac{D(s)}{D(s) + N(s)} \right) = \frac{D(0)}{D(0) + N(0)} = \text{finite value}$$

Ramp Input:

$$e_{ss} = \lim_{s \rightarrow 0} (s E(s)) = \lim_{s \rightarrow 0} \left(s \cdot \frac{E}{R} \cdot \frac{1}{s^2} \right) = \lim_{s \rightarrow 0} \left(\frac{D(s)}{s(D(s) + N(s))} \right) = \frac{D(0)}{0} = \text{infinite value}$$

Parabolic Input:

$$e_{ss} = \lim_{s \rightarrow 0} (s E(s)) = \lim_{s \rightarrow 0} \left(s \cdot \frac{E}{R} \cdot \frac{1}{s^3} \right) = \lim_{s \rightarrow 0} \left(\frac{D(s)}{s^2(D(s) + N(s))} \right) = \frac{D(0)}{0} = \text{infinite value}$$

Type “1” System

The steady-state error of a type “1” system to *step*, *ramp*, and *parabolic* inputs can be calculated as follows:

Step Input:

$$e_{ss} = \lim_{s \rightarrow 0} (s E(s)) = \lim_{s \rightarrow 0} \left(s \cdot \frac{E}{R} \cdot \frac{1}{s} \right) = \lim_{s \rightarrow 0} \left(\frac{s D(s)}{(s D(s)) + N(s)} \right) = \frac{0}{N(0)} = \text{zero}$$

Ramp Input:

$$e_{ss} = \lim_{s \rightarrow 0} (s E(s)) = \lim_{s \rightarrow 0} \left(s \cdot \frac{E}{R} \cdot \frac{1}{s^2} \right) = \lim_{s \rightarrow 0} \left(\frac{D(s)}{s D(s) + N(s)} \right) = \frac{D(0)}{N(0)} = \text{finite value}$$

Parabolic Input:

$$e_{ss} = \lim_{s \rightarrow 0} (s E(s)) = \lim_{s \rightarrow 0} \left(s \cdot \frac{E}{R} \cdot \frac{1}{s^3} \right) = \lim_{s \rightarrow 0} \left(\frac{D(s)}{s(s D(s) + N(s))} \right) = \frac{D(0)}{0} = \text{infinite value}$$

Summary: (following the pattern of the above calculations)

Type	Steady State System Error		
	Step	Ramp	Parabolic
0	finite	∞	∞
1	0	finite	∞
2	0	0	finite
3	0	0	0