Intermediate Dynamics
Partial Velocities and Partial Angular Velocities

(Reference: T. R. Kane and D. A. Levinson, ***Dynamics, Theory and Applications***, McGraw-Hill, 1985.)

Partial Velocities

If ${}^R\underset{\sim}{v}_P$ the ***velocity*** of some point P within a mechanical system can be written in terms of a set of ***generalized coordinates*** $q_i \ (i=1,\ldots,n)$ and their ***time derivatives*** $\dot{q}_i \ (i=1,\ldots,n)$, then the ***partial velocities*** of P are defined to be the ***partial derivatives*** of ${}^R\underset{\sim}{v}_P$ with respect to the $\dot{q}_i \ (i=1,\ldots,n)$. That is,

$$\boxed{\frac{\partial\, {}^R\underset{\sim}{v}_P}{\partial \dot{q}_i} \quad (i=1,\ldots,n)} \tag{1}$$

These vectors represent the ***changes*** in the ***velocity*** of P resulting from ***changes*** in each of $\dot{q}_i \ (i=1,\ldots,n)$ the time derivatives of the coordinates. It can be shown that they also represent the ***changes*** in $\underset{\sim}{r}_P$ the ***position*** of P resulting from ***changes*** each of $q_i \ (i=1,\ldots,n)$ the generalized coordinates. Note that, in general, ${}^R\underset{\sim}{v}_P$ can be written in terms of the partial velocities as follows.

$$\boxed{{}^R\underset{\sim}{v}_P = \sum_{i=1}^{n}\left(\frac{\partial \underset{\sim}{r}_P}{\partial q_i}\right)\dot{q}_i + \frac{\partial \underset{\sim}{r}_P}{\partial t} = \sum_{i=1}^{n}\left(\frac{\partial\, {}^R\underset{\sim}{v}_P}{\partial \dot{q}_i}\right)\dot{q}_i + \frac{\partial \underset{\sim}{r}_P}{\partial t}} \tag{2}$$

Here, the term $\dfrac{\partial \underset{\sim}{r}_P}{\partial t}$ accounts for position vectors that depend ***explicitly on time*** (e.g. some specified motion within the system).

Partial Angular Velocities

Similarly, if the ***angular velocity*** of a body B within a mechanical system can be written in terms of a set of ***generalized coordinates*** $q_i \ (i = 1,\ldots,n)$ and their ***time derivatives*** $\dot{q}_i \ (i=1,\ldots,n)$, then the ***partial angular velocities*** of B are defined to be the ***partial derivatives*** of ${}^R\underset{\sim}{\omega}_B$ with respect to the $\dot{q}_i \ (i=1,\ldots,n)$.

$$\boxed{\frac{\partial\, {}^R\underset{\sim}{\omega}_B}{\partial \dot{q}_i} \quad (i=1,\ldots,n)} \tag{3}$$

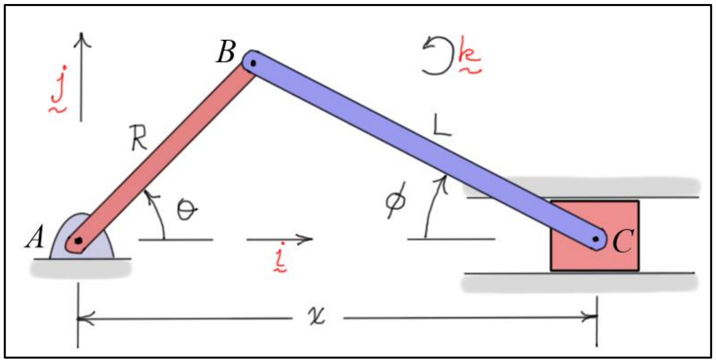
These vectors represent the ***changes*** in the ***angular velocity*** of a body resulting from ***changes*** in the $\dot{q}_i \ (i = 1, \ldots, n)$. In general, ${}^R\underset{\sim}{\omega}_B$ can be written in terms of the partial velocities as follows.

$$ {}^R\underset{\sim}{\omega}_B = \sum_{i=1}^{n} \left(\frac{\partial\, {}^R\underset{\sim}{\omega}_B}{\partial \dot{q}_i} \right) \dot{q}_i + ({}^R\underset{\sim}{\omega}_B)_t \tag{4} $$

Here, $\left({}^R\underset{\sim}{\omega}_B\right)_t$ is that part of the angular velocity vector that ***depends explicitly on time***.

<u>Example</u>

Consider the ***slider-crank mechanism*** shown in the diagram. It can be shown that the ***velocity*** of piston C and the ***angular velocity*** of connecting rod BC can be written as



$$ {}^R\underset{\sim}{v}_C = -R\left[\sin(\theta) + \left(\frac{\cos(\theta)\sin(\phi)}{\cos(\phi)} \right) \right] \dot{\theta}\, \underset{\sim}{i} $$

and

$$ {}^R\underset{\sim}{\omega}_{BC} = -\left(\frac{R\cos(\theta)}{L\cos(\phi)} \right) \dot{\theta}\, \underset{\sim}{k} $$

Using these expressions, the ***partial velocity*** of C and ***partial angular velocity*** of connecting rod BC with respect to $\dot{\theta}$ are easily identified to be as follows.

$$ \frac{\partial\, {}^R\underset{\sim}{v}_C}{\partial \dot{\theta}} = -R\left[\sin(\theta) + \left(\frac{\cos(\theta)\sin(\phi)}{\cos(\phi)} \right) \right] \underset{\sim}{i} \qquad \frac{\partial\, {}^R\underset{\sim}{\omega}_{BC}}{\partial \dot{\theta}} = -\left(\frac{R\cos(\theta)}{L\cos(\phi)} \right) \underset{\sim}{k} $$