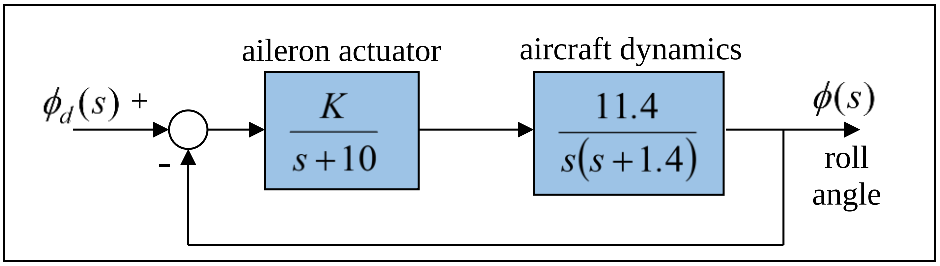
Introductory Control Systems
Performance Design Problem
Ref: Dorf & Bishop, Modern Control Systems, 12th edition, Prentice-Hall, Inc, 2010

Problem: The block diagram for roll control of a jet fighter is shown below. The goal is to select a suitable value of parameter K so the response of the system to a unit step command $\phi_d(t)$ will provide a response $\phi(t)$ that is a fast response with less than 16% overshoot.



Solution:
a) Using ***block diagram reduction***, the closed-loop transfer function is found to be

$$T(s) = \frac{\phi}{\phi_d}(s) = \frac{11.4K}{s(s+1.4)(s+10)+11.4K}$$

b) Expanding the denominator of $T(s)$ gives the ***characteristic equation***

$$s^3 + 11.4s^2 + 14s + 11.4K = 0$$

The ***poles*** of the system for three K values are shown in the table. Note over this range of values, the system is ***second-order dominant***. The real pole left of $s = -10$ is ***insignificant***.

K	*Poles*
0.7	$-10.09, -0.6545 \pm 0.602j$
3	$-10.368, -0.5161 \pm 1.7414j$
6	$-10.689, -0.3555 \pm 2.505j$

c) Given the insignificance of the real pole, the ***transfer function*** can be ***approximated*** as

$$T(s) = \frac{11.4K}{(s+p)(s^2+2\zeta\omega_n s+\omega_n^2)} \approx \frac{11.4K/p}{s^2+2\zeta\omega_n s+\omega_n^2}$$

The ***poles*** of the ***approximate second-order system*** can be written as

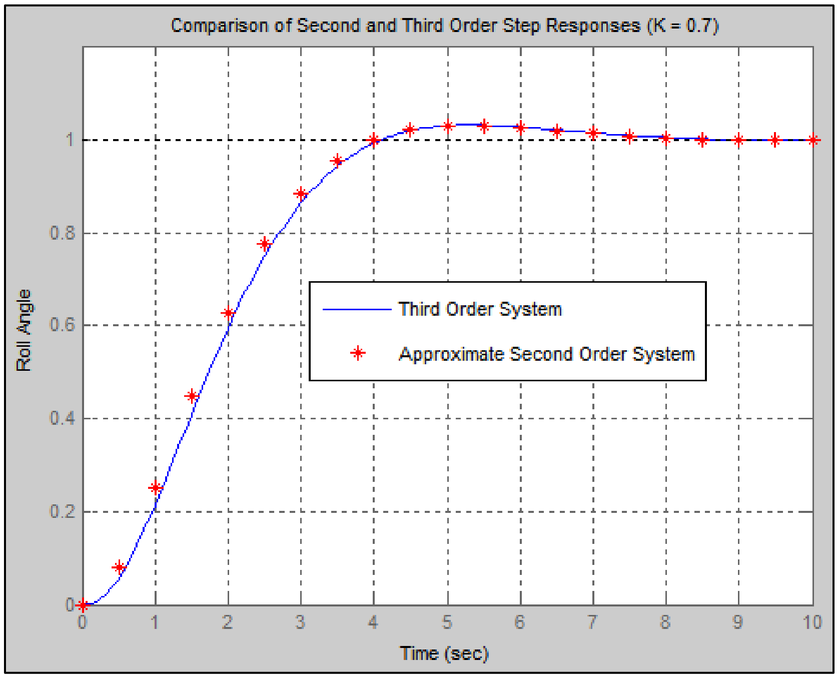
$$\sigma \pm j\omega = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} \quad \Rightarrow \quad \sigma = -\zeta\omega_n \quad \text{and} \quad \omega = \omega_n\sqrt{1-\zeta^2}$$

These equations can be solved for ω_n and ζ to get $\boxed{\omega_n = \sqrt{\sigma^2 + \omega^2}}$ and $\boxed{\zeta = -\sigma / \omega_n}$. Now, given the ***real*** and ***imaginary parts*** of the poles, the above two equations can be used to calculate ω_n and ζ. The table below lists these values for the three K values used above. It also lists the estimated percent overshoot and peak time discussed in earlier notes.

K	ω_n	ζ	$\%OS$	$T_p(s)$
0.7	0.8893	0.7360	3.3	5.21
3	1.816	0.284	39	1.80
6	2.53	0.14	64	1.25

d) The plot shows a sample result for $K = 0.7$. Note that the second-order approximation accurately predicts the response.

e) Using ***trial and error***, it can be shown that for $K = 1.275$, the damping ratio is $\zeta = 0.52$ for the approximate second-order system. This yields approximately a 16% overshoot.



f) What value of K yields an ITAE optimal response?

For a third-order system, the optimal form of characteristic equation for a step input is

$$\boxed{s^3 + 11.4s^2 + 14s + 11.4K = s^3 + 1.75\omega_n s^2 + 2.15\omega_n^2 s + \omega_n^3}$$

Comparing the coefficients of s^2 and s provides ***two different values*** for ω_n, so a single K value cannot be chosen using this approach. However, note that the system is ***approximately second order***, so $\zeta = 0.7$ gives ITAE optimal step response. Again, using trial and error, it can be shown that $K \approx 0.75$ gives $\zeta \approx 0.7$. As expected, the step response of this system is very similar to that shown in part (d).