Intermediate Dynamics
Principle of Virtual Work

Principle of Virtual Work

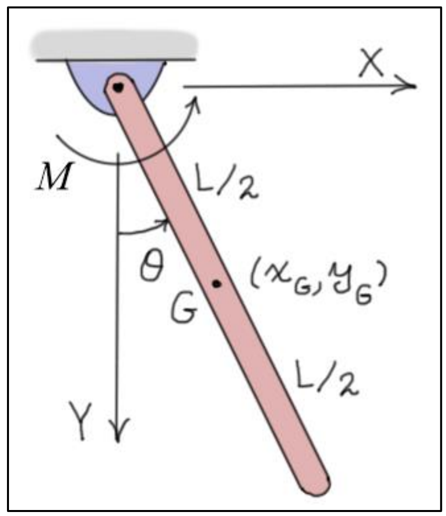
If a mechanical system whose ***configuration*** is defined by a set of ***independent generalized coordinates*** $q_k\ (k=1,...,n)$ is in ***static equilibrium***, then the ***generalized force*** associated with each of the generalized coordinates $q_k\ (k=1,...,n)$ must be ***zero***. That is,

$$\boxed{F_{q_k}=0} \quad (k = 1,\ldots,n)$$

Recall that if a system has "*n*" ***degrees of freedom***, then "*n*" independent generalized coordinates are required to completely define its configuration. Hence, for an "*n*" ***degree of freedom system*** in static equilibrium, "*n*" ***independent equilibrium equations*** can be written.

Example

For the ***simple pendulum*** shown, the ***torque*** *M* required to hold the bar at some ***constant*** angle θ can be calculated using the ***principle of virtual work***. To do this, note that the weight force and the applied torque are the only ***active forces*** and ***torques***. Using the angle θ as the generalized coordinate, the principle of virtual work can be written as follows.



$$0=F_\theta=\left(F_\theta\right)_{\underset{\sim}{W}}+\left(F_\theta\right)_{\underset{\sim}{M}}=\left(\underset{\sim}{W}\cdot\frac{\partial\, {}^R\underset{\sim}{v}_G}{\partial\dot{\theta}}\right)+\left(\underset{\sim}{M}\cdot\frac{\partial\, {}^R\underset{\sim}{\omega}_B}{\partial\dot{\theta}}\right)$$

$$=\left(mg\,\underset{\sim}{j}\cdot\frac{L}{2}\left(\cos(\theta)\,\underset{\sim}{i}-\sin(\theta)\,\underset{\sim}{j}\right)\right)+\left(-M\underset{\sim}{k}\right)\cdot\left(-\underset{\sim}{k}\right)$$

$$\Rightarrow \boxed{M=mg\frac{L}{2}\sin(\theta)}$$

Contrast this approach to ***summing moments*** on a free-body-diagram of the bar.

Note: The contribution of the ***weight force*** could also have been calculated using the ***potential energy*** function. Using the *X*-axis as the ***horizontal datum*** for the potential energy function, the ***generalized force*** associated with the weight force can be written as

$$\left(F_\theta\right)_{\underset{\sim}{W}}=-\frac{\partial}{\partial\theta}\left(-mg\frac{L}{2}\cos(\theta)\right)=-mg\frac{L}{2}\sin(\theta)$$