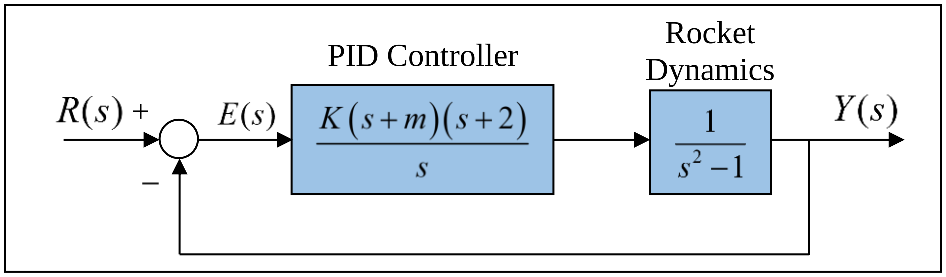
Introductory Control Systems
Stability Design Problem

Problem:

The block diagram for a rocket with a PID control system with two real zeros is given to be



Here, $R(s)$ and $Y(s)$ represent the ***desired*** and ***actual attitude angles*** of the rocket (assuming the rocket moves in a vertical plane). Note that the rocket dynamics are ***unstable***, because the poles are at ± 1.

a) Use the ***Routh-Hurwitz*** (RH) criterion to find the range of the parameters m and K so that the close-loop system is ***stable***.

b) ***Select*** the parameters so the ***steady-state error*** to a ramp input is less than 10%.

c) Find the ***percent overshoot*** to a step input for the design of part (b).

a) To apply the RH criterion to this system, find the ***closed-loop transfer function***, identify the ***characteristic equation***, and build the ***RH array***.

$$T(s) = \frac{K(s+m)(s+2)}{s(s^2-1) + K(s+m)(s+2)} = \frac{K(s+m)(s+2)}{s^3 + Ks^2 + [K(m+2)-1]s + 2mK}$$

RH Array:

$$\begin{array}{c|ccc} s^3 & 1 & K(m+2)-1 & 0 \\ s^2 & K & 2mK & 0 \\ s^1 & b_1 & b_2 & \\ s^0 & c_1 & & \end{array}$$

where

$$b_1 = \frac{-1}{K}\begin{vmatrix} 1 & K(m+2)-1 \\ K & 2mK \end{vmatrix} = \frac{-1}{\cancel{K}}\left[2m\cancel{K} - \cancel{K}\left(K(m+2)-1\right)\right] = K(m+2) - (2m+1)$$

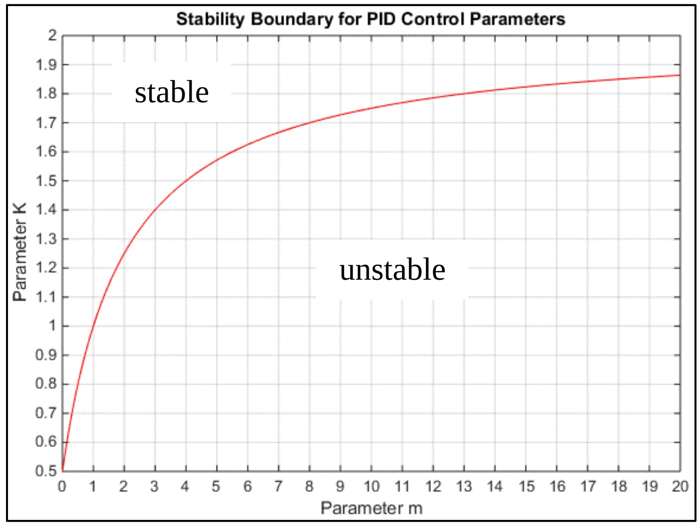
$$b_2 = \frac{-1}{K}\begin{vmatrix} 1 & 0 \\ K & 0 \end{vmatrix} = 0 \qquad c_1 = \frac{-1}{b_1}\begin{vmatrix} K & 2mK \\ b_1 & 0 \end{vmatrix} = \frac{-1}{b_1}\left[-b_1(2mK)\right] = 2mK$$

Stability Requirements:

All elements of the ***first column*** must have the ***same algebraic sign***, which leads to the following results.

$\boxed{K > 0}$, $\boxed{m > 0}$, $\boxed{K > (2m+1)/(m+2)}$

See the plot at the right for the stable and unstable regions. The ***red line*** indicates where $K = (2m+1)/(m+2)$ and, hence, indicates the stability boundary.



b) To satisfy the ***error requirement***, first find the ***error transfer function***. If $E(s)$ is the system output, then $G = 1$ and $H = \dfrac{K(s+m)(s+2)}{s(s^2-1)}$, and the error transfer function is

$$\boxed{\frac{E}{R}(s) = \frac{G}{1+GH} = \frac{1}{1+H} = \frac{1}{1+N_H/D_H} = \frac{D_H}{D_H+N_H} = \frac{s(s^2-1)}{s(s^2-1)+K(s+m)(s+2)}}$$

Steady State Error to a Ramp Input:

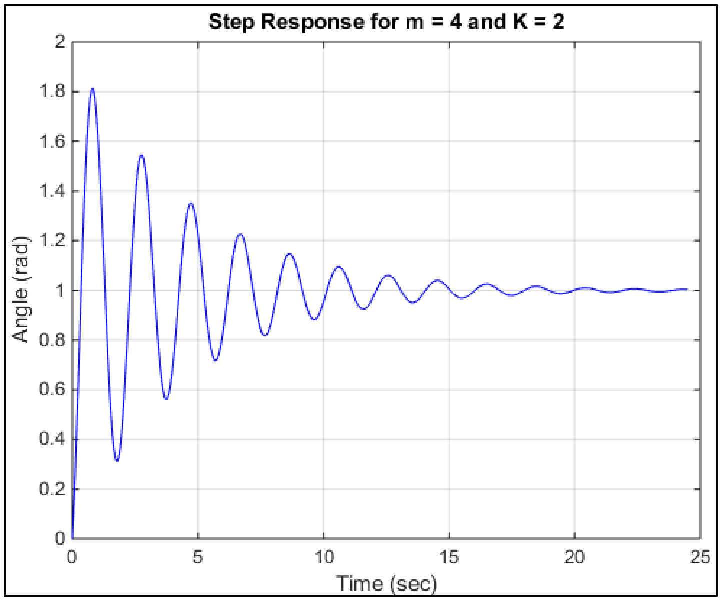
$$\boxed{e_{ss} = \lim_{s\to 0}\left[s\frac{1}{s^2}\frac{E}{R}(s)\right] = \lim_{s\to 0}\left[\frac{1}{\not{s}}\left(\frac{\not{s}(s^2-1)}{s(s^2-1)+K(s+m)(s+2)}\right)\right] = \frac{-1}{2mK}}$$

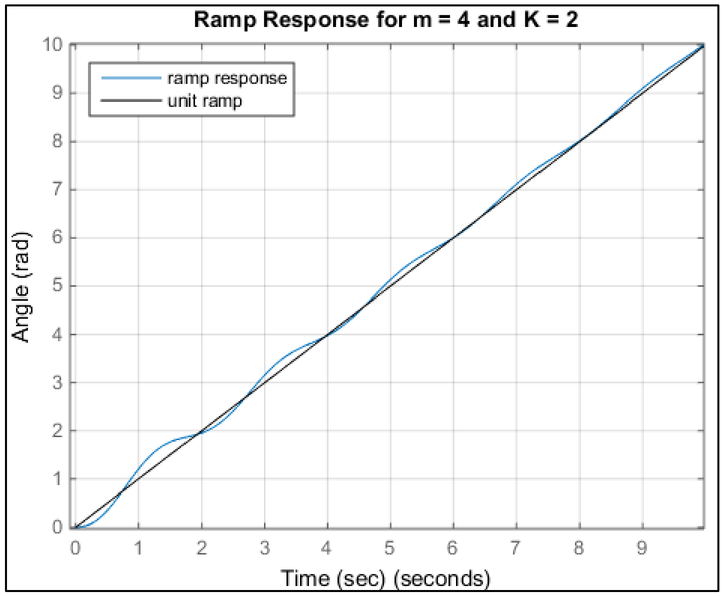
To satisfy the requirement, set the ***absolute value*** of e_{ss} to be less than 0.1.

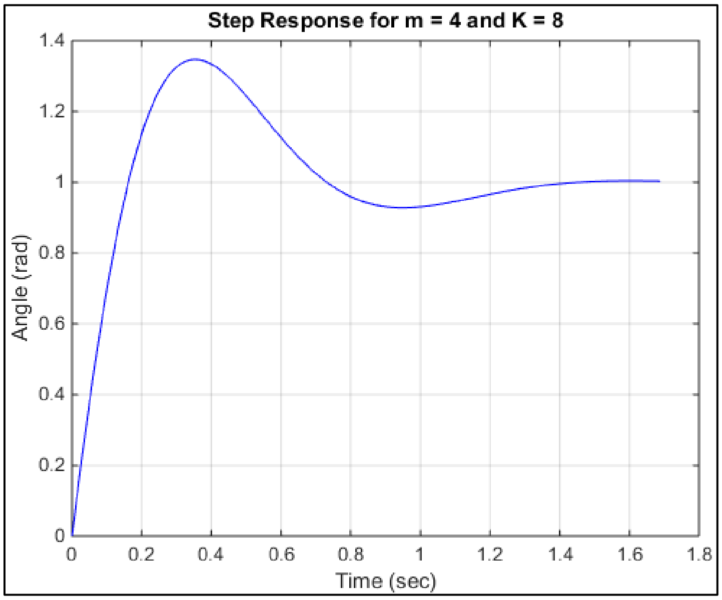
$$\boxed{\frac{1}{2mK} < 0.1} \quad \Rightarrow \boxed{mK > 5}$$

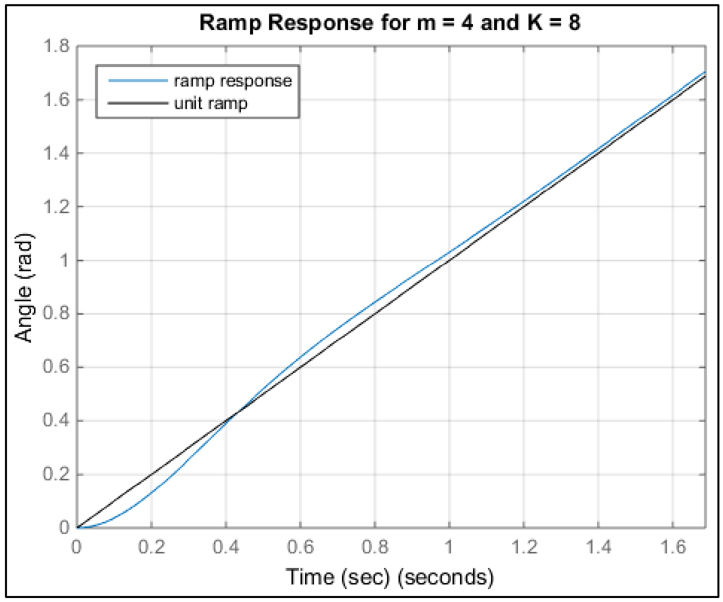
The requirement can be satisfied for many values of m and K, e.g., $m = 4$ and $K = 2$.

c) Using MATLAB, the response of the system for ***various values*** of the parameters can be explored. The plots below show the ***step*** and ***ramp*** responses for two cases: 1) $m = 4$, $K = 2$, and 2) $m = 4$, $K = 8$. The percent overshoot for case 1 is 81%, and the percent overshoot for case 2 is 35%. The ramp response for case 1 is close to the unit ramp but is ***oscillatory*** well passed 10 seconds. The ramp response for case 2 is also close to the unit ramp, and it settles into a ***steady-state ramp*** in about 1.3 seconds.


Step Response for m = 4 and K = 2
Angle (rad)
Time (sec)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
5
10
15
20
25


Ramp Response for m = 4 and K = 2
ramp response
unit ramp
Angle (rad)
Time (sec) (seconds)
0
1
2
3
4
5
6
7
8
9
10
0
1
2
3
4
5
6
7
8
9


Step Response for m = 4 and K = 8
Angle (rad)
Time (sec)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8


Ramp Response for m = 4 and K = 8
ramp response
unit ramp
Angle (rad)
Time (sec) (seconds)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6