Intermediate Dynamics
Introduction to Lagrangian Dynamics

Newton/Euler Equations of Motion

One approach to finding the ***equations of motion*** (EOM) of a mechanical system is to use the ***Newton/Euler*** equations of motion.

$$\sum_i \underset{\sim}{F}_i = m\,{}^R\underset{\sim}{a}_G$$

$$\sum_i \left(\underset{\sim}{M}_G\right)_i = \left(\underset{\approx}{I}_G \cdot {}^R\underset{\sim}{\alpha}_B\right) + \left({}^R\underset{\sim}{\omega}_B \times \underset{\sim}{H}_G\right)$$

or

$$\sum_i \left(\underset{\sim}{M}_A\right)_i = \left(\underset{\approx}{I}_G \cdot {}^R\underset{\sim}{\alpha}_B\right) + \left({}^R\underset{\sim}{\omega}_B \times \underset{\sim}{H}_G\right) + \left(\underset{\sim}{r}_{G/A} \times m\,{}^R\underset{\sim}{a}_G\right)$$

Here, G is the ***mass center*** of the body and A is ***any point***. In this approach, ***bodies are isolated*** one-by-one using ***free body diagrams***. Then the above equations are used to write the EOM. As a result, these equations contain ***unknown constraint forces*** and ***moments***. Hence, the differential equations of motion contain algebraic unknowns. Equations of this form are often referred to as ***differential/algebraic equations*** of motion.

Lagrange's Equations of Motion

The application of ***Lagrange's equations of motion*** differs from the application of the Newton/Euler equations in the following ways:

- Focus is on the ***entire system*** rather than individual components.
- EOM are formulated in terms of the ***scalar functions*** of ***work*** and ***kinetic energy***.
- ***Constraint forces*** and ***moments*** that do ***no work*** are ***eliminated*** from the analysis.

For many systems the resulting equations of motion form a set of ***differential*** equations. For more complex systems they form a set of ***differential/algebraic*** equations. The specific form of Lagrange's equations is presented in later notes.

Note:

Lagrange's equations are not the only ***system-based*** formulation of EOM. Other system-based formulations include ***d'Alembert's Principle*** and ***Kane's Equations***.