

## Introductory Control Systems

### What is a Root Locus Diagram?

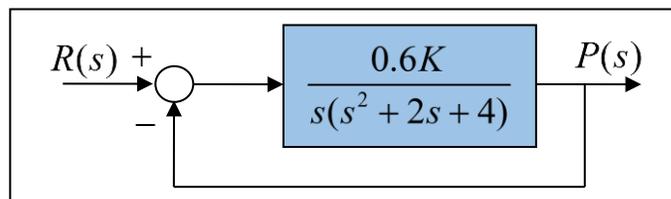
Definition: A *root locus diagram* is the plot of the *paths* of the poles of a closed loop system as a single parameter  $K$  is varied.

Root Locus Diagram:  $0 \leq K < +\infty$

Complementary Root Locus Diagram:  $-\infty < K \leq 0$

Example:

- *Proportional position control* of a *space platform* can be expressed by the block diagram below. The open-loop platform dynamics is *second order* and *under-damped*.



Question: How do the *poles* of this closed-loop system *vary* as the parameter  $K$  changes?

- One way to answer this question is simply to find the closed loop transfer function, identify the *characteristic equation*, and compute the *roots* for a variety of parameter values. For this system, the characteristic equation is  $s^3 + 2s^2 + 4s + 0.6K = 0$ .
- The figure below shows a *plot* of the *poles* of the system for a given set of  $K$  values. The red stars indicate the locations of the poles for  $1 \leq K \leq 27$  with an increment  $\Delta K = 2$ . The blue circles indicate the locations of the poles for  $-1 \leq K \leq -27$  with an increment of  $\Delta K = -2$ .

Observations about this root locus diagram:

- As  $K$  *increases* (becoming more *positive*), the *real pole* moves from the *origin* along the *negative real axis*, while a pair of *complex conjugate poles* crosses over the imaginary axis and move into the *right-half plane*. As  $K$  *decreases* (becoming more negative), the *real pole* moves from the origin along the *positive real axis*, while a pair of *complex conjugate poles* move further into the *left-half plane*.

- As  $K$  increases toward  $+\infty$  or decreases toward  $-\infty$  the branches follow *asymptotes*.
- The poles *do not* necessarily *move uniformly* along the branches.
- Although the poles do not move uniformly along the branches, the branches are *smooth curves*.
- Recall that if all the poles of a system are in the *left-half plane*, it is *stable*, and if any of the poles are in the *right-half plane*, it is *unstable*. This system is stable for  $0 \leq K \leq 13$  (approximately). A *more precise estimate* of the upper limit of  $K$  can be found using the Routh-Hurwitz method or by specifying smaller increments in MATLAB.

