Intermediate Dynamics
Lagrange's Equations for Multi-Degree-of-Freedom Systems

The ***configuration*** of systems with N degrees-of-freedom (DOF) can be defined in terms of N ***generalized coordinates***, say $q_k \; (k=1,\ldots,N)$. The ***differential equations of motion*** of the system can be derived using ***Lagrange's equations*** as defined in Eq. (1).

$$\boxed{\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_k}\right) - \frac{\partial K}{\partial q_k} = F_{q_k}} \quad (k=1,\ldots,N) \qquad (1)$$

Here, (…with NB representing the number of bodies in the system)

$$K = \tfrac{1}{2}\sum_{j=1}^{NB}\left\{ m_j \left({}^R\underset{\sim}{v}_{G_j}\right)^2 + {}^R\underset{\sim}{\omega}_{B_j} \cdot \underset{\sim}{H}_{G_j} \right\}$$ …the ***kinetic energy*** of the system

$F_{q_k} =$ generalized force associated with generalized coordinate q_k

(due to ***all*** the forces and torques acting on the system)

Note:

It is important that the kinetic energy K and the generalized forces $F_{q_k} \; \left(k=1,\ldots,N\right)$ be written ***only in terms*** of $q_k \; \left(k=1,\ldots,N\right)$, $\dot{q}_k \; \left(k=1,\ldots,N\right)$, and ***no other variables***.

If some of the forces and torques are ***conservative***, their contributions to the equations of motion can be calculated in terms of ***potential energy functions***. In this case, the differential equations of motion can be derived using the form of Lagrange's equations given in Eq. (2).

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = \left(F_{q_k}\right)_{nc}} \quad (k=1,\ldots,N) \qquad (2)$$

Here, $L = K - V$ is the ***Lagrangian*** of the system, V is the ***potential energy function*** for the ***conservative forces*** and ***torques***, and $\left(F_{q_k}\right)_{nc}$ is the ***generalized force*** associated with q_k for the ***nonconservative forces*** and ***torques***, only.