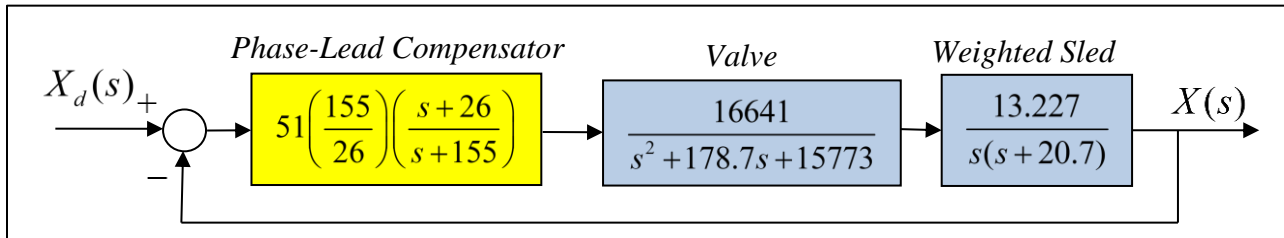


## Introductory Motion and Control

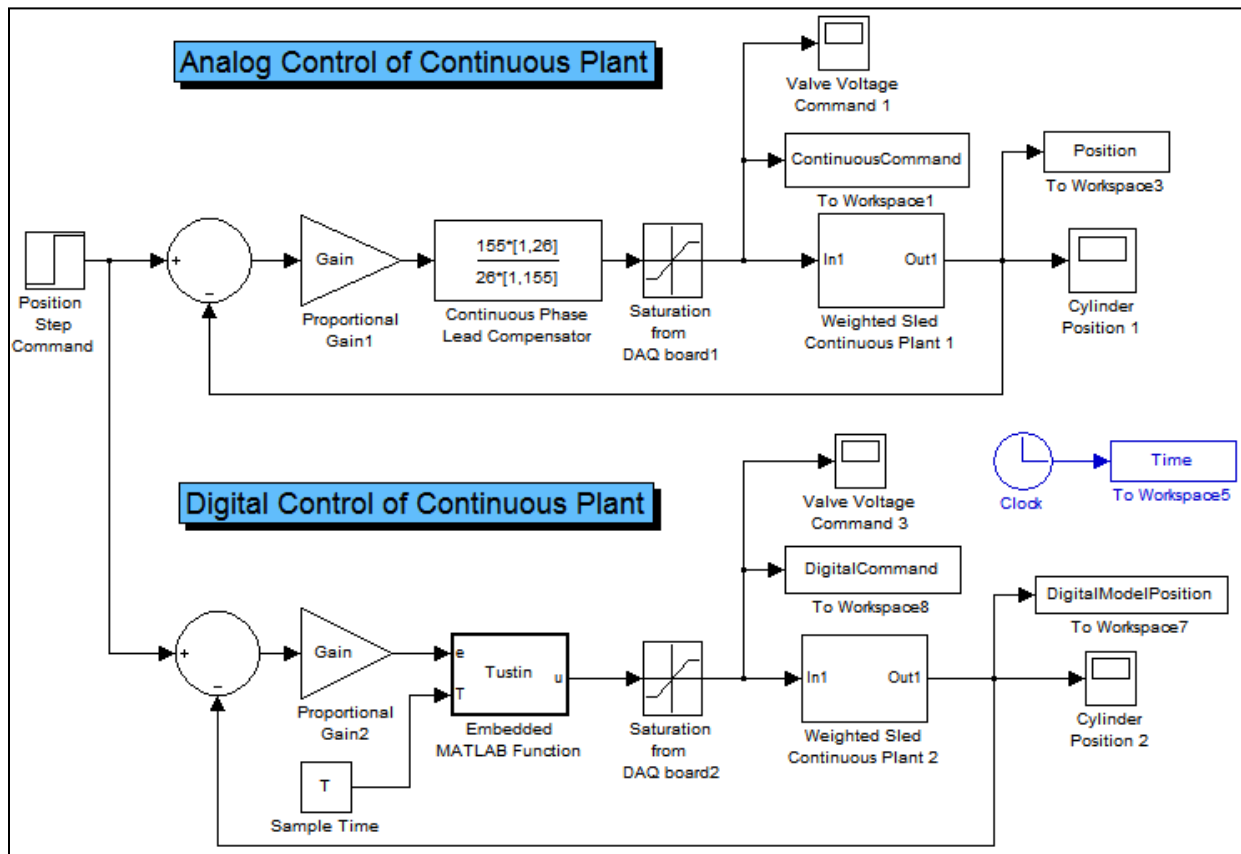
### Example: Digital Phase-Lead Control of a Weighted Sled

Consider *position control* of a hydraulically-driven weighted sled system using a *continuous* phase-lead compensator as illustrated in the diagram.



Closed-Loop Position Control of a Weighted Sled System

In the Simulink model below, the system is modelled using both *continuous* and *discrete* compensators. The *embedded MATLAB function* allows the discrete compensator to be implemented directly as a *difference equation*. The *coefficients* of the *difference equation* depend on the *sampling time*  $T$ . The difference equation used here is the same equation that would be coded into the digital controlling software. (e.g. *LabVIEW code*) The effects of *saturation* and *dead-band* are also included.



The **difference equation** coded in the embedded MATLAB function was developed from the continuous compensator using **Tustin's** approximation. Using  $\alpha = 155/26$ ,

$$G_c(s) = \frac{U(s)}{E(s)} = \alpha \left[ \frac{s+26}{s+155} \right] = \alpha \left[ \frac{\frac{2}{T} \left[ \frac{z-1}{z+1} \right] + 26}{\frac{2}{T} \left[ \frac{z-1}{z+1} \right] + 155} \right]$$

After some algebraic manipulation, this equation can be reduced to

$$U(z) = \alpha \left[ \frac{26T+2}{155T+2} \right] E(z) + \alpha \left[ \frac{26T-2}{155T+2} \right] z^{-1} E(z) - \left[ \frac{155T-2}{155T+2} \right] z^{-1} U(z)$$

This is equivalent to the **difference equation**

$$u(k) = \alpha \left[ \frac{26T+2}{155T+2} \right] e(k) + \alpha \left[ \frac{26T-2}{155T+2} \right] e(k-1) - \left[ \frac{155T-2}{155T+2} \right] u(k-1)$$

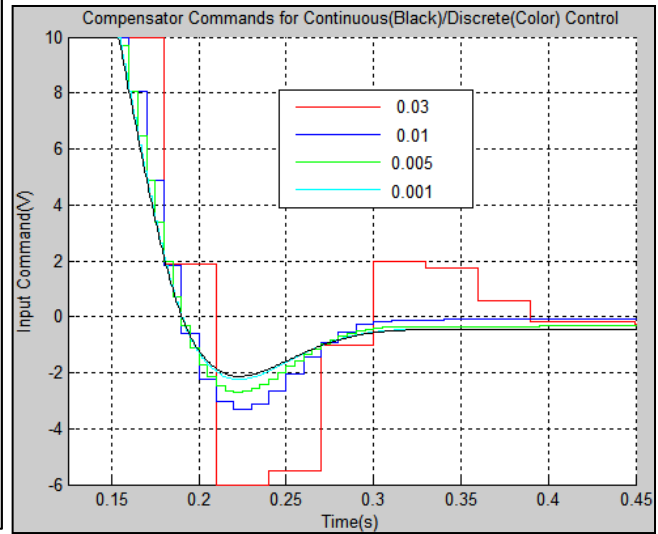
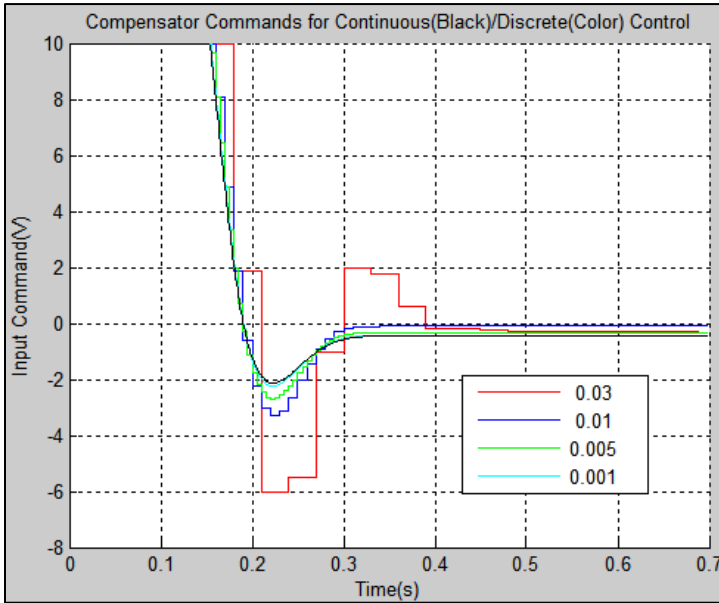
Here,  $e(k)$  and  $u(k)$  represent the error and control signal values at the **present time**, and  $e(k-1)$  and  $u(k-1)$  represent the signal values at the **previous time step**.

The figures below show results for the **compensator command** and the **sled position** response for continuous and a series of discrete compensators with

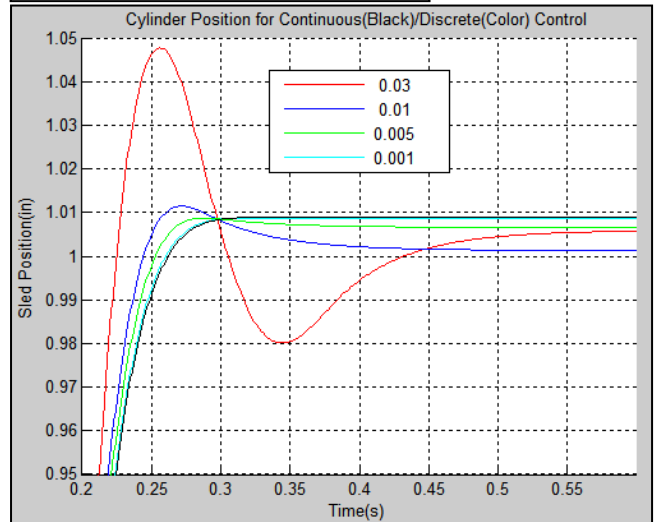
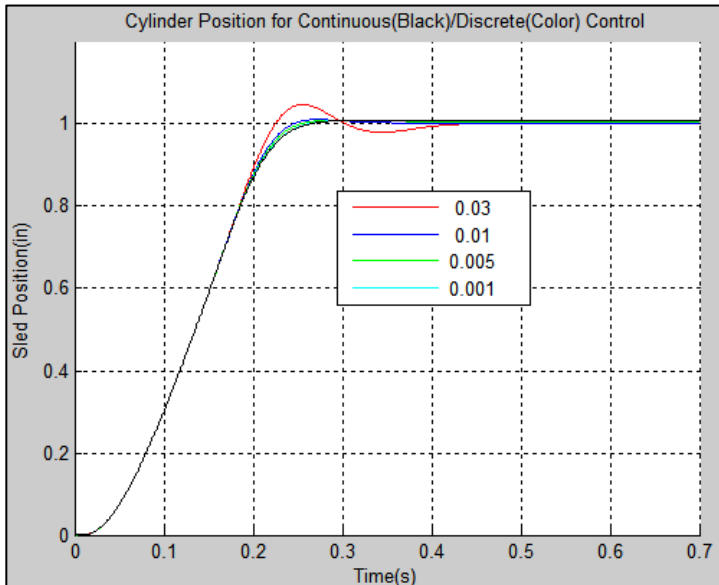
<b>Variable</b>	<b>Value</b>
Gain	51
Desired Position	1 (in)
Sample Times	(0.001, 0.005, 0.01, 0.03) (sec)

The **Bode diagram** of the **closed loop system** is also shown, indicating a closed loop **bandwidth** of approximately  $60 \text{ (rad/s)} \approx 9.55 \text{ (Hz)}$ . For good results, the sample rate should be approximately  $f \approx 20(9.55) = 191 \text{ (Hz)}$ . This corresponds to a sample time of  $T = 0.005 \text{ (sec)}$ .

## Compensator Commands



## Sled Position Response



## Closed Loop Frequency Response

