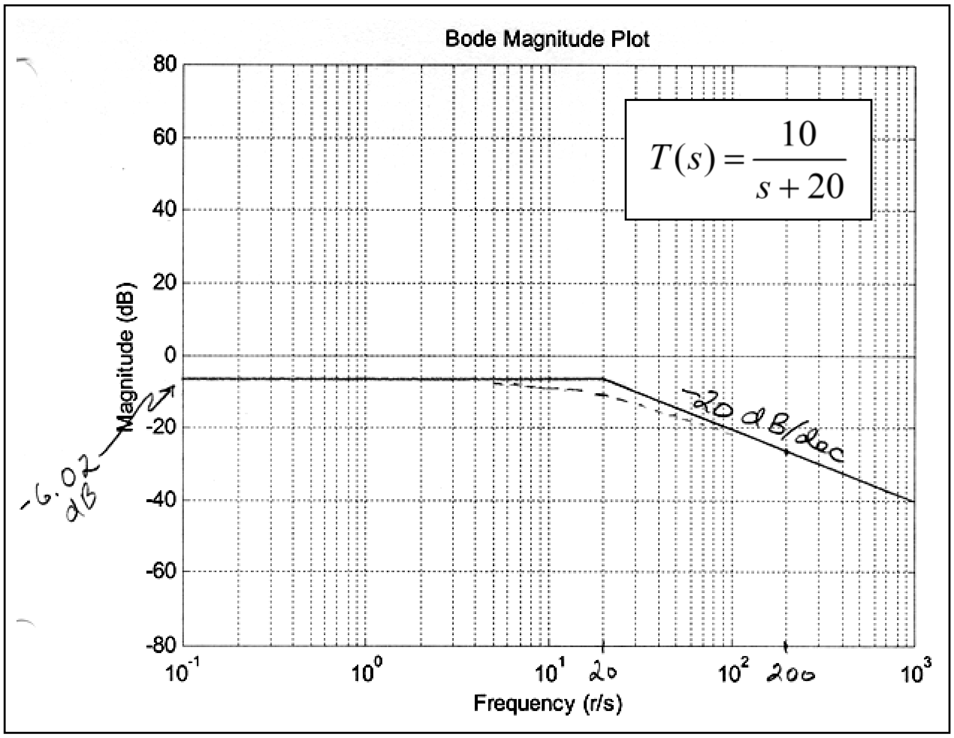
Introductory Control Systems
Bode Diagram Examples

Example 1:

The system is ***first-order*** with a gain $K = 10$ and a real pole at $s = -20$. Because the system has ***no poles*** or ***zeros*** at the ***origin***, the ***low-frequency asymptote*** is ***constant***. The system has a ***corner frequency*** at 20 (rad/s) at which point the slope ***decreases*** by 20 (dB/decade). The ***dotted line*** indicates the system is ***less responsive*** near the corner frequency than is indicated by the asymptotes. The system has ***zero*** phase at low frequencies and transitions to -90 (deg) as the corner frequency is passed.



<u>Corner Frequency</u>: $\omega = 20$ (rad/sec)

<u>Low frequency asymptote</u>: $\omega \ll 20$ (rad/sec)

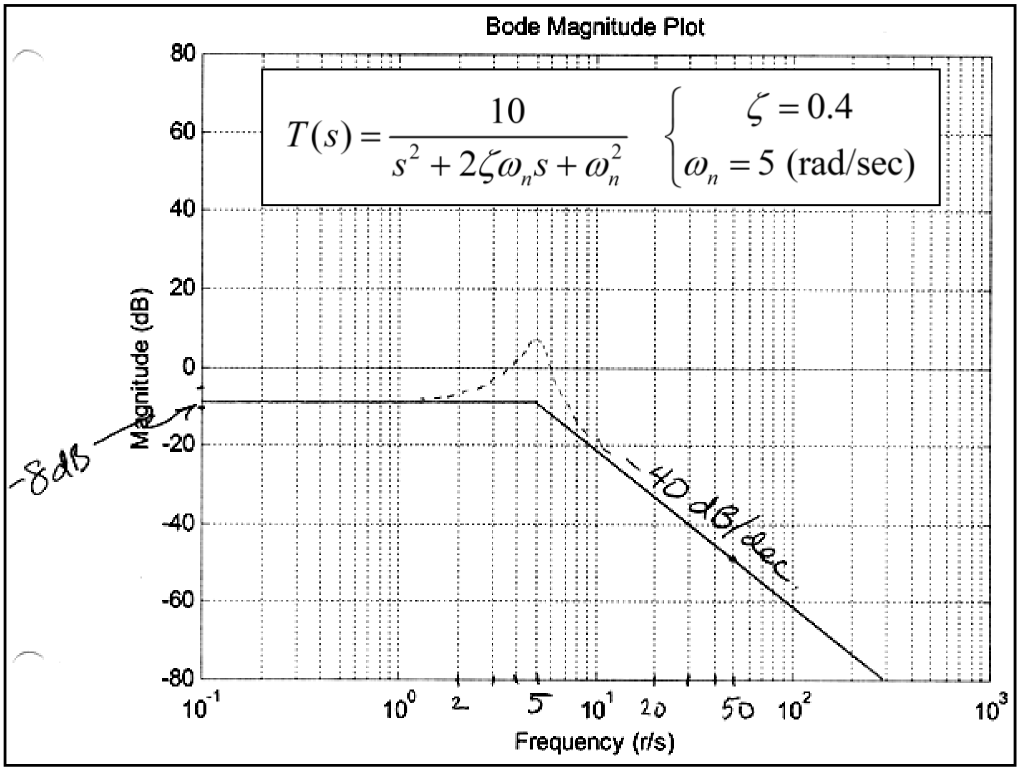
$$M(\omega) = \text{constant} = 20\log(10) - 20\log(20) = -6.02 \text{ (dB)}$$
$$\phi = 0 \text{ (deg)}$$

<u>High frequency asymptote</u>: $\omega \gg 20$ (rad/sec)

$$M(\omega) = -20\log(\omega) \;\ldots\; (\text{a } -20 \text{ dB/decade slope})$$
$$\phi = -90 \text{ (deg)}$$

Example 2:

The system is ***second-order*** with a gain $K = 10$ and a pair of complex poles with natural frequency $\omega_n = 5$ (rad/s) and damping ratio $\zeta = 0.4$. Because the system has ***no poles*** or ***zeros*** at the ***origin***, the ***low-frequency asymptote*** is ***constant***. The system has a ***corner frequency*** at $\omega = 5$ (rad/s) at which point the slope ***decreases*** by 40 (dB/decade). The ***dotted line*** indicates the system is ***more responsive*** near the corner frequency than is indicated by the asymptotes. The system has ***zero*** phase at low frequencies and transitions to -180 (deg) as the corner frequency is passed.



<u>Corner Frequency</u>: $\omega = 5$ (rad/sec)

<u>Low frequency asymptote</u>: $\omega \ll 5$ (rad/sec)

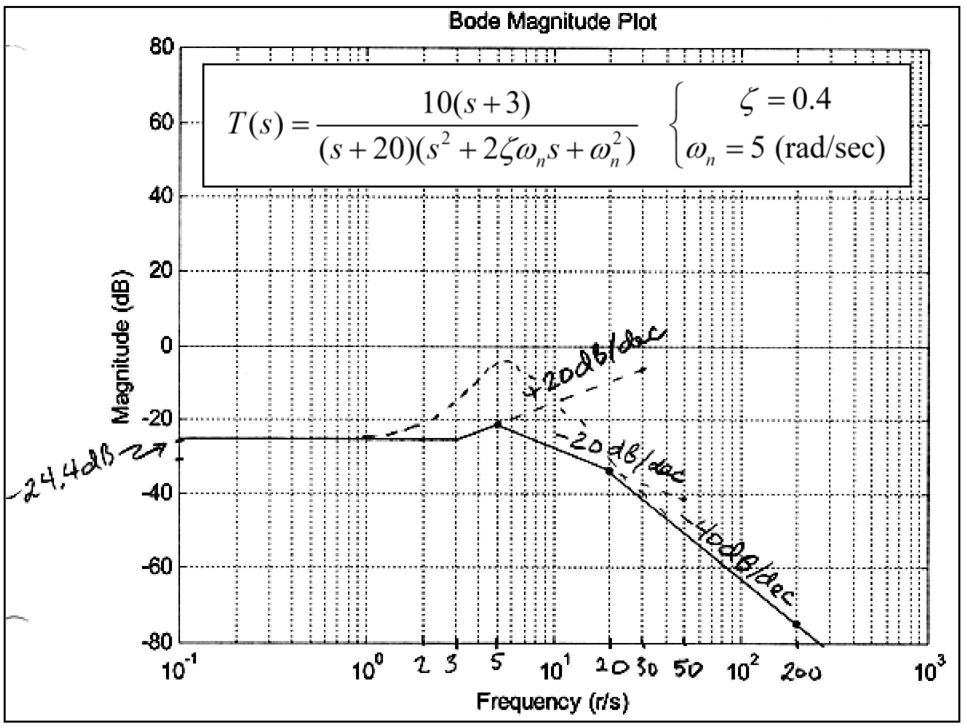
$$M(\omega) = \text{constant} = 20\log(10) - 40\log(5) = -7.96 \text{ (dB)} \approx -8 \text{ (dB)}$$
$$\phi = 0 \text{ (deg)}$$

<u>High frequency asymptote</u>: $\omega \gg 5$ (rad/sec)

$$M(\omega) = -40\log(\omega) \ \ldots \text{ (a } -40 \text{ dB/decade slope)}$$
$$\phi = -180 \text{ (deg)}$$

Example 3:

The system is ***third-order*** with a gain $K = 10$, a real zero at $s = -3$, a pair of complex poles with natural frequency $\omega_n = 5$ (rad/s) and damping ratio $\zeta = 0.4$, and a real pole at $s = -20$. Because the system has ***no poles*** or ***zeros*** at the ***origin***, the ***low-frequency asymptote*** is ***constant***. The system has ***corner frequencies*** at $\omega = 3, 5, 20$ (rad/s). At 3 (rad/s) the ***slope increases*** by 20 (dB/decade), at 5 (rad/s) it ***decreases*** by 40 (dB/decade), and at 20 (rad/s) it ***decreases*** by another 20 (dB/decade). The ***dotted line*** indicates the system is ***more responsive*** near $\omega_n = 5$ (rad/s) than is indicated by the asymptotes. The response at this frequency is increased by the presence of the zero. The system has ***zero*** phase at low frequencies and transitions to -90 (deg) and -180 (deg) as the corner frequencies are passed.



Corner Frequencies: $\omega = 3, 5, 20$ (rad/sec)

Low frequency asymptote: $\omega \ll 3$ (rad/sec)

$$M(\omega) = \text{constant} = 20\log(10) + 20\log(3) - 20\log(20) - 40\log(5) = -24.4 \text{ (dB)}$$
$$\phi = 0 \text{ (deg)}$$

$3 < \omega < 5$ (rad/sec): $M(\omega)$ has a $+20$ dB/decade slope

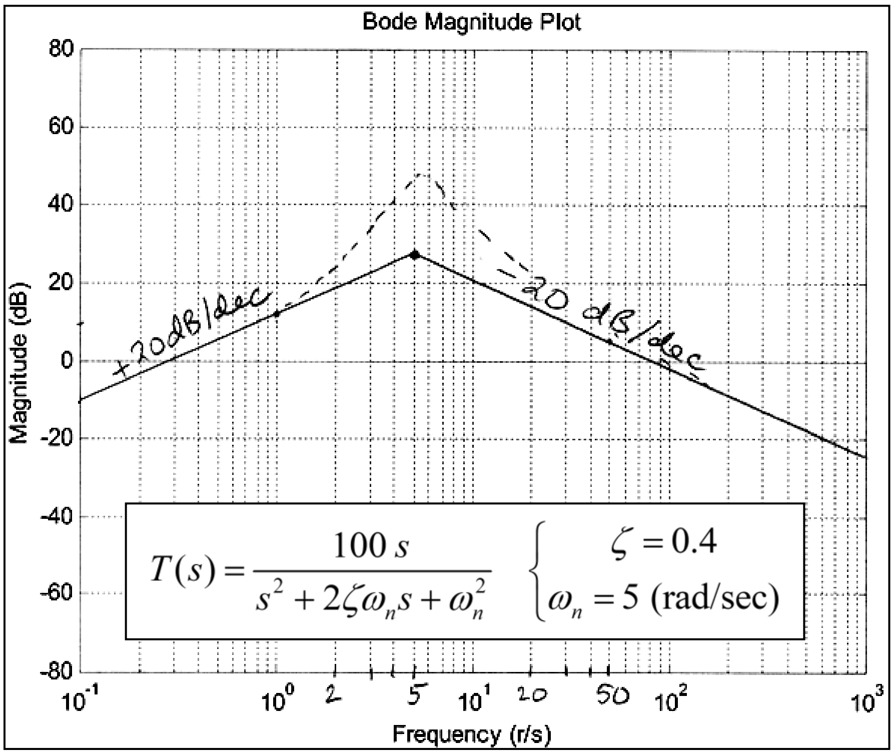
$5 < \omega < 20$ (rad/sec): $M(\omega)$ has a -20 dB/decade slope

High frequency asymptote: $\omega \gg 20$ (rad/sec)

$$M(\omega) \text{ has a } -40 \text{ dB/decade slope}$$
$$\phi = -180 \text{ (deg)}$$

Example 4:

The system is ***second-order*** with a gain $K = 100$, a ***zero*** of ***order one*** at the ***origin***, and a ***pair*** of ***complex poles*** with natural frequency $\omega_n = 5$ (rad/s) and damping ratio $\zeta = 0.4$. Because the system has a ***zero*** of ***order one*** at the ***origin***, the ***low-frequency asymptote*** has a slope of $+20$ (dB/decade). The system has a ***corner frequency*** at $\omega = 5$ (rad/s) at which point the slope ***decreases*** by 40 (dB/decade). The ***dotted line*** indicates the system is ***more responsive*** near the corner frequency than is indicated by the asymptotes. The system has $+90$ (deg) phase at low frequencies and transitions to -90 (deg) as the corner frequency is passed.



Corner Frequency: $\omega = 5$ (rad/sec)

@ $\omega = 1$ (rad/sec): $M(\omega)\big|_{\omega=1} = 20\log(100) + 20\log(1) - 40\log(5) = 12$ (dB)

$\omega \ll 5$ (rad/sec):

$M(\omega)$ has a slope of $+20$ dB/decade due to the zero at the origin.
$\phi = 90$ (deg)

High frequency asymptote: $\omega \gg 20$ (rad/sec)

$M(\omega)$ has a -20 dB/decade slope
$\phi = -90$ (deg)

MATLAB® Results:

For comparison with the above results, the following diagrams were generated for each of the example systems using MATLAB®.