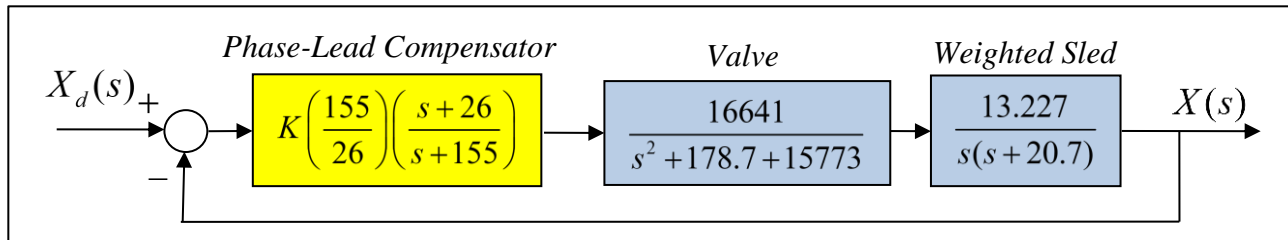


## Introductory Motion and Control

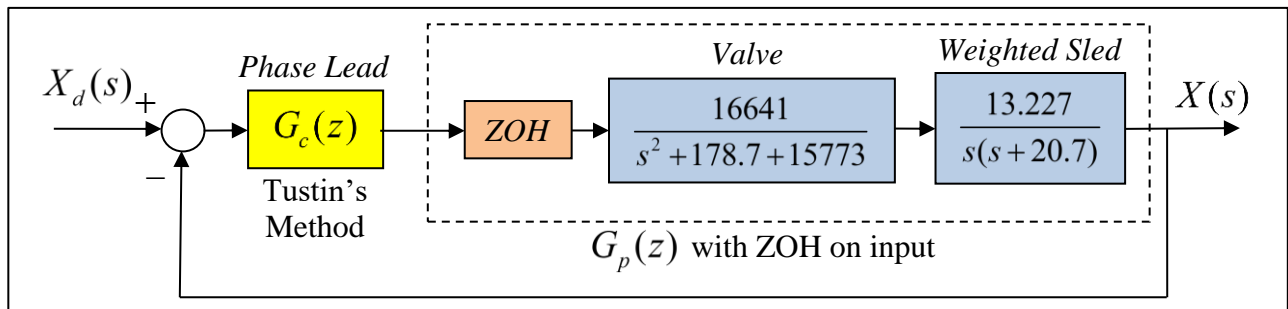
### Discrete Root Locus Analysis of a Weighted Sled System

Consider *position control* of a weighted sled system using a *continuous* phase-lead compensator as shown in the diagram below.



Closed-Loop Position Control of a Weighted Sled System

To determine the *effects* of *discretization* on the stability and *performance* of this system, a *discrete root locus analysis* can be done. *Discrete equivalents* for the *valve* and *weighted sled* transfer functions are found by assuming there is a *zero-order hold* on the valve input. Tustin's method is used here to find a discrete equivalent for the phase-lead compensator.



Discrete Closed-Loop Position Control of a Weighted Sled System

Using the “*c2d*” function in MATLAB and a *sample time* of  $T = 0.01$  (sec), the following results are obtained for the *compensator* and *valve/sled* transfer functions.

```

z = 26; p = 155;
numGc = p*[1,z]; denGc = z*[1,p]; sysGc = tf(numGc,denGc);

sysGcD = c2d(sysGc,T,'tustin')

Transfer function:
  3.795 z - 2.922
  -----
  z - 0.1268

Sampling time: 0.01
    
```

```

numGv= 16641.6; denGv= [1,178.688,15772.5]; sysGv= tf(numGv,denGv)

numGs = 13.227; denGs = [1,20.7,0]; sysGs = tf(numGs,denGs)

sysGp = series(sysGv,sysGs); sysGpD = c2d(sysGp,T,'zoh')

Transfer function:
  6.103e-005 z^3 + 0.0004431 z^2 + 0.000294 z + 1.832e-005
  -----
  z^4 - 2.333 z^3 + 1.923 z^2 - 0.7263 z + 0.1362

Sampling time: 0.01

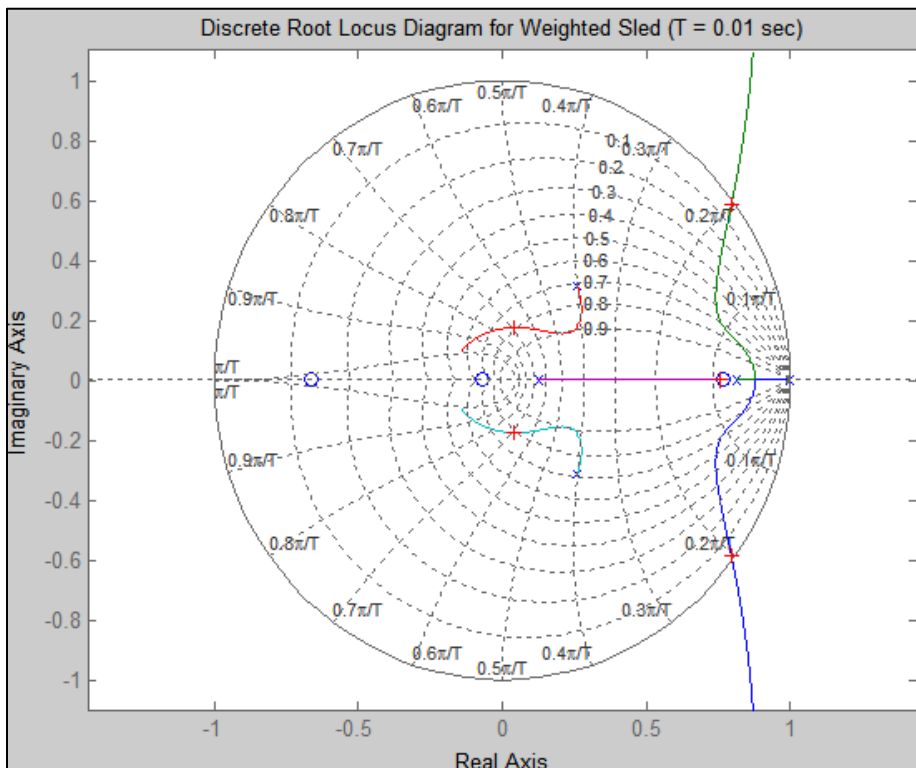
```

Given  $G_c(z)$  and  $G_p(z)$  the discretized transfer functions of the compensator and the valve/sled plant, the **discrete closed loop transfer function** can be computed.

$$\frac{X}{X_d}(z) = \frac{G_c(z)G_p(z)}{1 + G_c(z)G_p(z)}$$

As with continuous systems, the **characteristic equation** of the closed loop system with negative unity feedback is  $1 + G_c(z)G_p(z) = 0$ .

Using the “**rlocus**” and “**rlocfind**” commands, the **limiting gain** for stability is  $K \approx 125$ .



Note that the **stability boundary** is now the unit circle.

Contrast these results to the root locus diagram for the continuous system shown below. Here the *limiting gain* is found to be  $K \approx 187$ .

