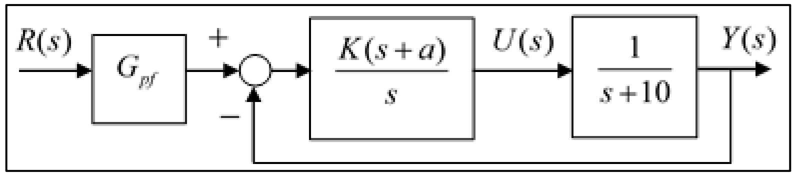
Introductory Motion and Control
Exercises #3 Pre-filters, ITAE Optimal Response, Control Effort

1. In this system a "PI" compensator and a pre-filter are used to control a 1st order plant. $U(s)$ is the control signal.



a) Design the "PI" compensator and the pre-filter so the closed loop system has ITAE optimal response to a step input with a settling time $T_s < 0.5 \text{ (sec)}$. b) Using the compensator and pre-filter you designed in part (a), find $u_{\max}$ and u_{ss} the maximum and steady-state values of the control signal $U(s)$ for a unit step input.

Answers: (for $\omega_n = 12 \text{ (rad/s)}$)

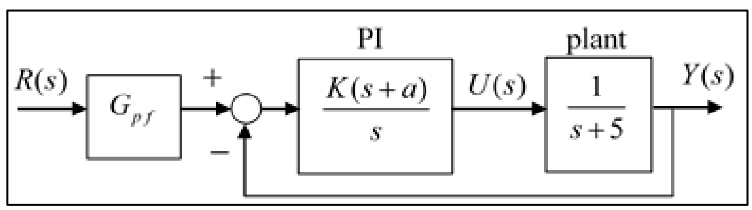
a) $K \approx 6.8$, $a \approx 21.18$, $G_{pf} = \dfrac{144}{6.8(s+21.18)}$; b) $\dfrac{U}{R}(s) = \dfrac{144(s+10)}{s^2+16.8s+144}$ (second order, under-damped, $\zeta \approx 0.7$, $\beta \approx 1.19$), $u_{ss} \approx 10$, $u_{\max} \approx 1.15 \times 10 = 11.5$ (15% overshoot)

2. Repeat problem (1) for a settling time $T_s < 0.1 \text{ (sec)}$.

Answers: (for $\omega_n = 60 \text{ (rad/sec)}$)

a) $K \approx 74$, $a \approx 48.65$, $G_{pf} = \dfrac{3600}{74(s+48.65)}$; b) $\dfrac{U}{R}(s) = \dfrac{3600(s+10)}{s^2+84s+3600}$ (second order, under-damped, $\zeta \approx 0.7$, $\beta \approx 0.238$, $u_{ss} = 10$, $u_{\max} \approx 30$ (over 200% overshoot)

3. In the system shown, a "PI" compensator and a pre-filter G_{pf} are used to control a 1st order plant. $U(s)$ is the control signal.



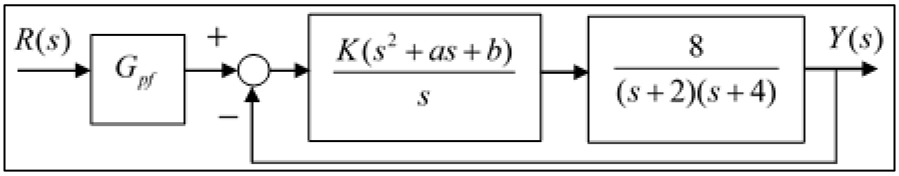
a) Design the "PI" compensator and the pre-filter so the closed loop system has ITAE optimal response to a unit step input $\left(R(s) = 1/s\right)$ with a settling time $T_s \le 0.4 \text{ (sec)}$. b) Using the compensator and pre-filter you designed in part (a), find the transfer function $U(s)/R(s)$ and estimate u_{ss} the steady-state value of the control signal $U(s)$ for a unit step input ($R(s) = 1/s$). c) How would you change the settling time specified in part (a) if you wanted to lower $u_{\max}$ the maximum value of $u(t)$ the input to the plant?

Answers: (for $\omega_n = 15 \text{ (rad/s)}$)

a) $\zeta = 0.7$, $G_c(s) = \dfrac{16(s+14.06)}{s}$, $G_{pf}(s) = \dfrac{225}{16(s+14.06)}$; b) $\dfrac{U}{R}(s) = \dfrac{225(s+5)}{s^2+21s+225}$ and $u_{ss} = 5$

c) Increase the settling time requirement so the plant is not driven as hard

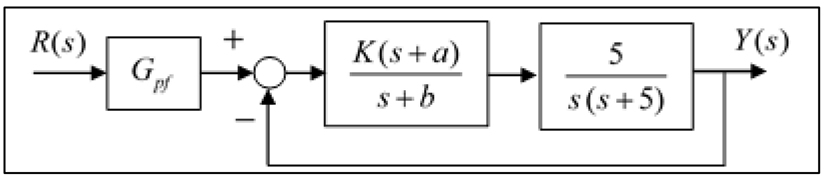
4. In the system shown, a "PID" compensator and a pre-filter are used to control a 2nd order plant. Design the "PID" compensator and the pre-filter so the closed loop system has ITAE optimal response to a step input with a settling time $T_s < 0.5$ (sec).



Answers:

For $\omega_n = 15$ (rad/s): $K \approx 2.53$, $a \approx 23.5$, $b \approx 167$, $G_{pf} = \dfrac{15^3}{8K\left(s^2 + a\,s + b\right)}$

5. In the system shown, a compensator and a pre-filter are used to control a 2nd order plant. Design the compensator and the pre-filter so the closed loop system has ITAE optimal response to a step input with a settling time $T_s < 0.5$ (sec).



Answers:

For $\omega_n = 15$ (rad/s): $K \approx 75.5$, $a \approx 8.94$, $b \approx 21.25$ (phase-lead compensator), $G_{pf} = \dfrac{15^3}{5K\left(s + a\right)}$