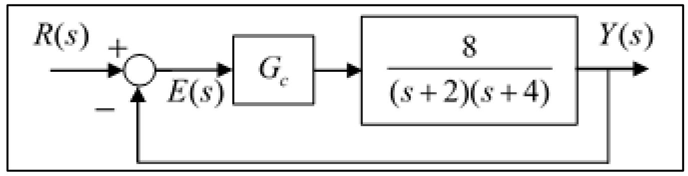
Introductory Motion and Control
Exercises #4 Phase-lead Compensator Design Using Bode Diagrams

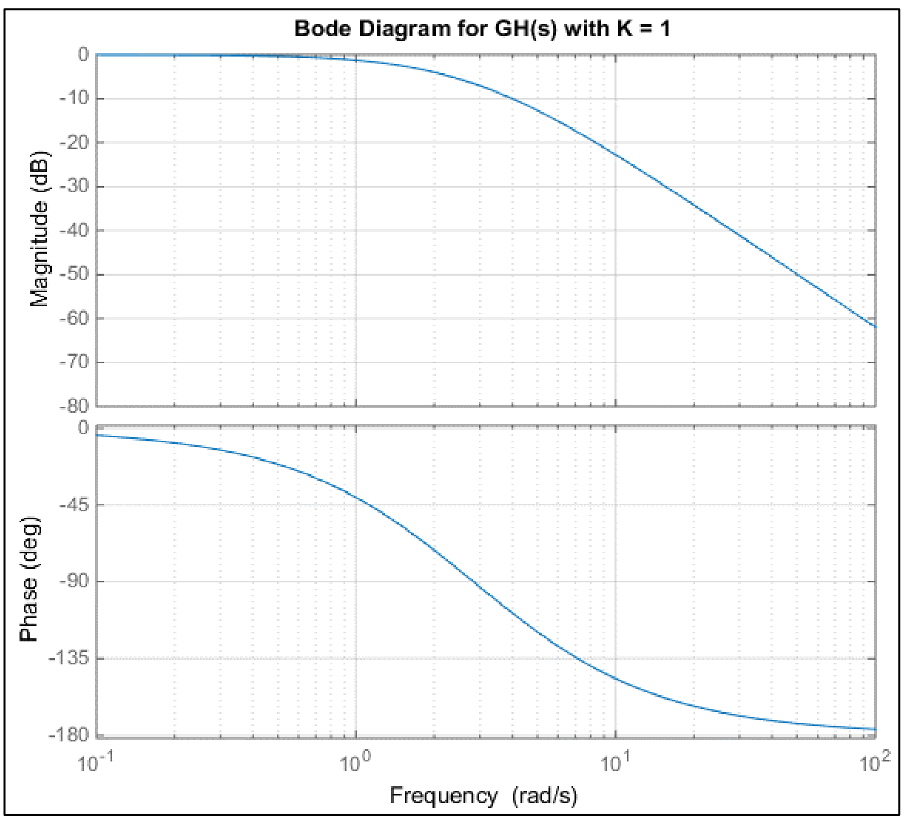


1. The block diagram shows a second-order system that is to be controlled by a phase-lead compensator $G_c(s)$. a) Assuming $G_c(s)=K$, find the range of K values required to make e_{ss} the steady state error due to a unit step input less than 0.01. b) Estimate PM the phase margin of the system using a K value that satisfies the condition of part (a). The Bode diagram of $GH(s)$ with $G_c(s)=K=1$ is shown below. c) Design a phase-lead compensator that will increase the phase margin of the system by 30 (deg).

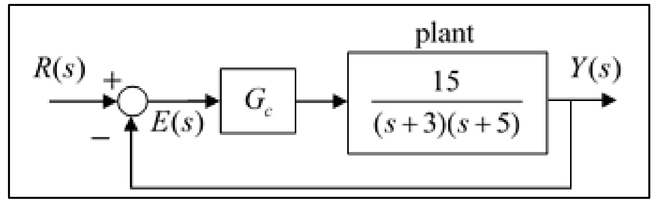
Answers:

a) $K>99$; b) Using diagram with $K=100$, $PM\approx 15$ (deg); c) $G_c\left(s\right)\approx 300\left[\frac{s+23.09}{s+69.28}\right]$

Note: The actual phase margin for $K=100$ is approximately 12 degrees. The estimated compensated system yields a phase margin of 39.4 degrees. Need further iteration to achieve desired phase margin increase.



2. The block diagram shows a second-order system to be controlled by a phase-lead compensator $G_c(s)$. a) Assuming $G_c(s)=K$, find the range of K values required to make e_{ss} the steady state error due to a unit step input less than 0.04. b) Estimate PM the phase margin of the system using a K value that satisfies the condition of part (a). The Bode diagram of $GH(s)$ with $G_c(s)=K=1$ is shown below. c) Design a phase lead compensator that will increase the phase margin of the system by $30\ (\text{deg})$. d) Sketch a root locus diagram for the closed loop system using the compensator you designed in part (c). Estimate the settling time of the closed loop system to a step input for large values of K.

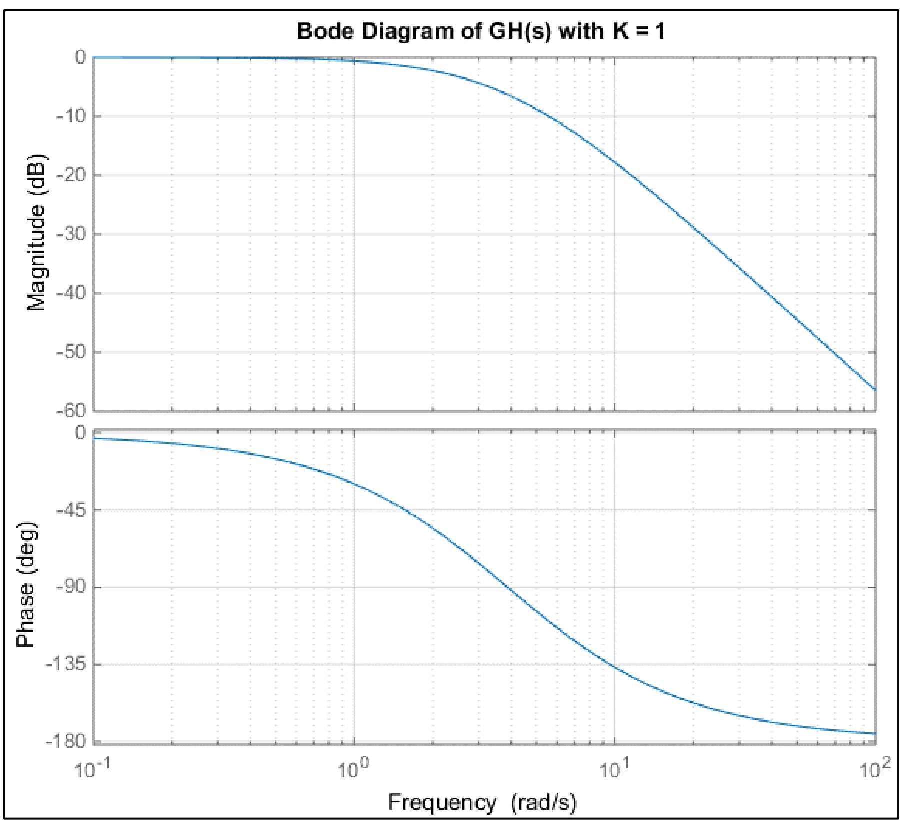


Answers:

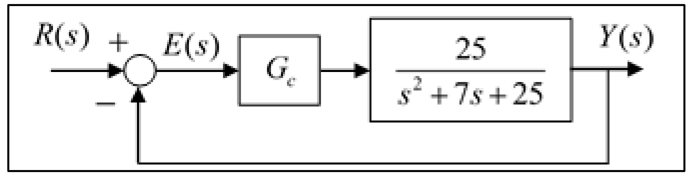
a) $K>24$; b) Using the diagram with $K=30$, $PM\approx 23\ (\text{deg})$; c) $G_c(s)\approx 90\left[\frac{s+17.32}{s+51.96}\right]$

d) For large values of K, there will be a near-pole zero cancellation with two under-damped poles approaching vertical asymptotes at $s=-21.34$. The settling time of these poles is $T_s\approx 0.19\ (\text{sec})$.

Note: The actual phase margin with $K=30$ is approximately 22 degrees. The estimated compensated system yields a phase margin of 46.8 degrees. Need further iteration to achieve desired phase margin increase.



3. The block diagram shows an under-damped second order system to be controlled by a phase-lead compensator $G_c(s)$. a) Assuming $G_c(s) = K$, find the range of K values required to make e_{ss} the steady state error due to a unit step input less than 0.06. b) Estimate PM the phase margin of the system using a K value that satisfies the condition of part (a). The Bode diagram of $GH(s)$ with $G_c(s) = K = 1$ is shown below. c) Using the Bode diagram, design a phase lead compensator that will increase the phase margin of the system by $30\ (\text{deg})$. Use a gain K that will satisfy the condition of part (a). d) Sketch a root locus diagram for the closed loop system using the compensator you designed in part (c). Estimate the settling time of the closed loop system to a step input for large K values.

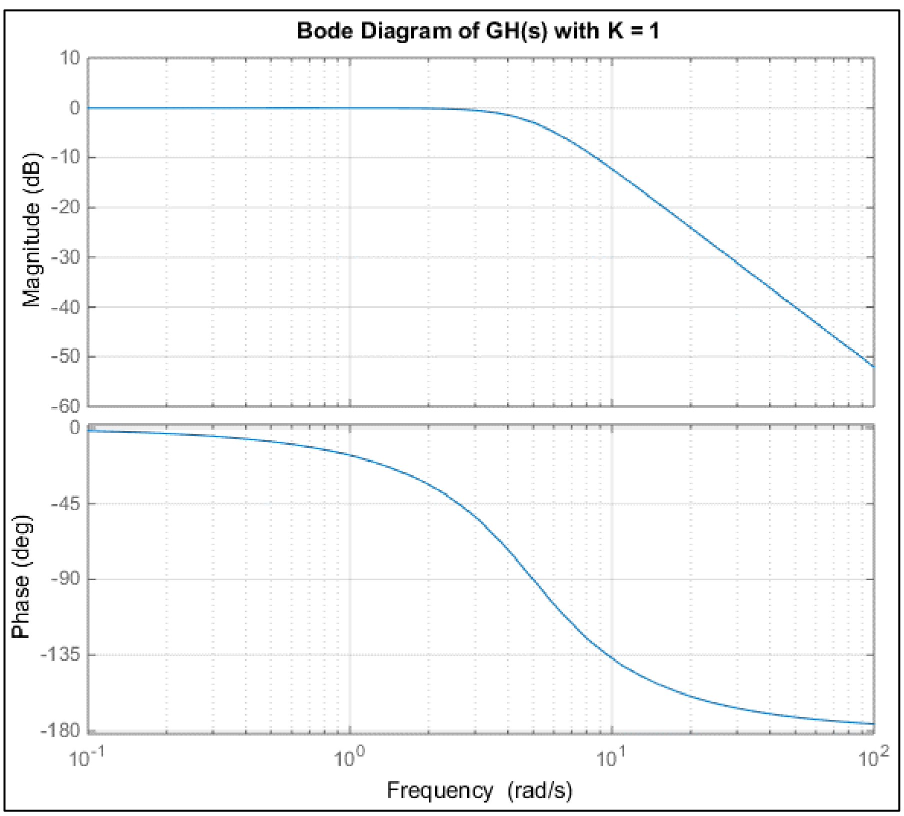


Answers:

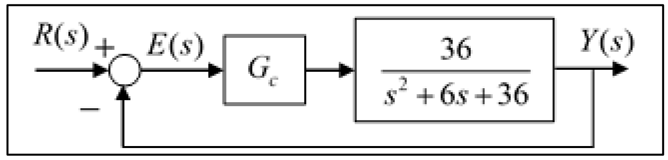
a) $e_{ss} = \dfrac{1}{K+1}$ and $K > 15.7$; b) Using $K = 17$, $PM \approx 20\ (\text{deg})$; c) $G_c(s) \approx 17(3)\left[\dfrac{s+15.6}{s+46.8}\right]$

d) For large values of K, there will be a near-pole zero cancellation with two under-damped poles approaching vertical asymptotes at $\sigma_A = -19.1$, so the settling time approaches $T_s \approx 4/19.1 = 0.21\ (\text{s})$.

Note: The actual phase margin with $K = 17$ is approximately 19.8 degrees. The estimated compensated system yields a phase margin of 44.9 degrees. Need further iteration to achieve desired phase margin increase.



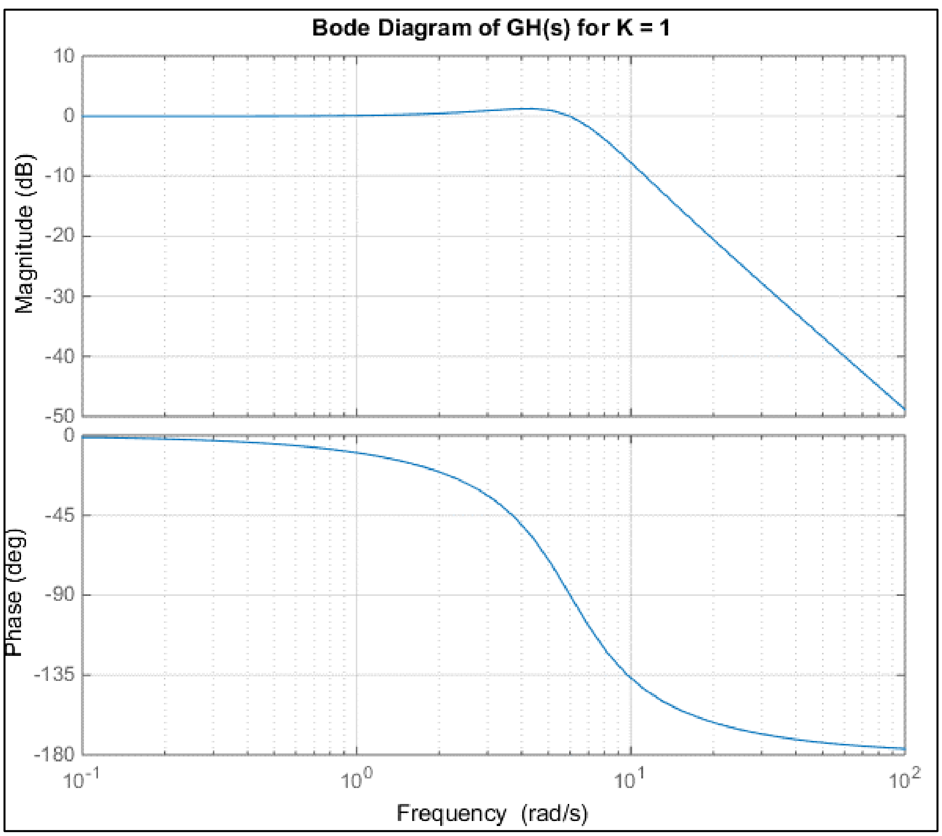
4. The block diagram shows a second order system to be controlled by a phase-lead compensator $G_c(s)$. a) Assuming $G_c(s) = K$, find the range of K values required to make e_{ss} the steady state error due to a unit step input less than or equal to 0.1. b) Estimate PM the phase margin of the system using a K value that satisfies the condition of part (a). The Bode diagram of $GH(s)$ with $G_c(s) = K = 1$ is shown below. c) Using the Bode diagram, design a phase lead compensator that will increase the phase margin of the system by 32 (deg). Use a gain K that will satisfy the condition of part (a).



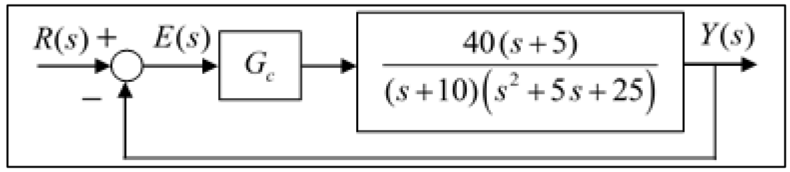
Answers:

a) $K \geq 9$; b) Using $K = 10$, $PM \approx 20$ (deg); c) $G_c(s) = 10(3.255)\left[\dfrac{s+13.86}{s+45.1}\right]$

Note: The actual phase margin with $K = 10$ is approximately 18.9 degrees. The estimated compensated system yields a phase margin of 45.6 degrees. Need further iteration to achieve desired phase margin increase.



5. The block diagram shows a third order system to be controlled by a phase-lead compensator $G_c(s)$. a) Assuming $G_c(s) = K$, find the range of K values required to make e_{ss} the steady state error due to a unit step input less than 0.05. b) Estimate PM the phase margin of the system using a K value that satisfies the condition of part (a). The Bode diagram of $GH(s)$ with $G_c(s) = K = 1$ is shown below. c) Using the Bode diagram, design a phase lead compensator that will increase the phase margin of the system by $30\ (\text{deg})$. Use a gain K that will satisfy the condition of part (a).



Answers:

a) $K \geq 23.75$; b) Using $K = 25$, $PM \approx 18\ (\text{deg})$; c) $G_c(s) = 25(3)\left[\dfrac{s+24.25}{s+72.75}\right]$

Note: The actual phase margin with $K = 25$ is approximately 18 degrees. The estimated compensated system yields a phase margin of 43.8 degrees. Need further iteration to achieve desired phase margin increase.