Elementary Dynamics – Equation Sheet #3

Linear Impulse and Momentum (for Systems of Particles)

$$\sum m_i\left(\underset{\sim}{v}_i\right)_1 + \sum\int_{t_1}^{t_2} \underset{\sim}{F}_i dt = \sum m_i\left(\underset{\sim}{v}_i\right)_2$$

or

$$m\left(\underset{\sim}{v}_G\right)_1 + \sum\int_{t_1}^{t_2} \underset{\sim}{F}_i dt = m\left(\underset{\sim}{v}_G\right)_2$$

$$\underset{\sim}{F}_{avg} = \frac{1}{\Delta t}\sum\int_{t_1}^{t_2} \underset{\sim}{F}_i dt = \frac{1}{\Delta t}\underset{\sim}{I}_{1\to 2}$$

Direct Central Impact of Two Particles

$$m_A\left(v_{Ax}\right)_1 + m_B\left(v_{Bx}\right)_1 = m_A\left(v_{Ax}\right)_2 + m_B\left(v_{Bx}\right)_2$$

$$e = \frac{\left(v_{Bx}\right)_2 - \left(v_{Ax}\right)_2}{\left(v_{Ax}\right)_1 - \left(v_{Bx}\right)_1}$$

Oblique Central Impact of Two Particles

$\left(v_{Ay}\right)_2 = \left(v_{Ay}\right)_1$ and $\left(v_{By}\right)_2 = \left(v_{By}\right)_1$

$$m_A\left(v_{Ax}\right)_1 + m_B\left(v_{Bx}\right)_1 = m_A\left(v_{Ax}\right)_2 + m_B\left(v_{Bx}\right)_2$$

$$e = \frac{\left(v_{Bx}\right)_2 - \left(v_{Ax}\right)_2}{\left(v_{Ax}\right)_1 - \left(v_{Bx}\right)_1}$$

Conservation of Linear Momentum

$$\sum\left(m_i\underset{\sim}{v}_i\right)_1 = \sum\left(m_i\underset{\sim}{v}_i\right)_2$$

$$\left(\sum m_i\right)\underset{\sim}{v}_G = \sum m_i\underset{\sim}{v}_i = \text{constant}$$

Collisions with a Fixed Surface

$\left(v_{Ay}\right)_2 = \left(v_{Ay}\right)_1$ and $\left(v_{Ax}\right)_2 = -e\left(v_{Ax}\right)_1$

Rigid Body Fixed Axis Rotation (2D)

$\underset{\sim}{\omega} = \omega\underset{\sim}{k} = \dot{\theta}\underset{\sim}{k}$ and $\underset{\sim}{\alpha} = \alpha\underset{\sim}{k} = \dot{\omega}\underset{\sim}{k} = \ddot{\theta}\underset{\sim}{k}$

$$\underset{\sim}{v}_P = \underset{\sim}{\omega}\times\underset{\sim}{r}_P = r\dot{\theta}\underset{\sim}{e}_\theta$$

$$\underset{\sim}{a}_P = \frac{d}{dt}\left(\underset{\sim}{\omega}\times\underset{\sim}{r}_P\right) = \left(\underset{\sim}{\alpha}\times\underset{\sim}{r}_P\right) + \underset{\sim}{\omega}\times\left(\underset{\sim}{\omega}\times\underset{\sim}{r}_P\right) = -r\dot{\theta}^2\underset{\sim}{e}_r + r\ddot{\theta}\underset{\sim}{e}_\theta$$

Constant Angular Acceleration

$$\alpha = \alpha_c = \text{ constant}$$

$$\omega = \omega_0 + \alpha_c t$$

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha_c t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

Relative Motion of Two Points of a Rigid Body (2D)

$$\underset{\sim}{v}_A = \underset{\sim}{v}_B + \underset{\sim}{v}_{A/B} = \underset{\sim}{v}_B + \left(\underset{\sim}{\omega}\times\underset{\sim}{r}_{A/B}\right)$$

$$\underset{\sim}{a}_A = \underset{\sim}{a}_B + \underset{\sim}{a}_{A/B} = \underset{\sim}{a}_B + \left(\underset{\sim}{\alpha}\times\underset{\sim}{r}_{A/B}\right) - \omega^2\underset{\sim}{r}_{A/B}$$

Point Moving on a Rigid Body: Rotating Axes

$$\underset{\sim}{v}_A = \underset{\sim}{v}_B + \underset{\sim}{v}_{Arel} + \underset{\sim}{\omega}\times\underset{\sim}{r}_{A/B} = \underset{\sim}{v}_B + (\underset{\sim}{v}_{A/B})_{xyz} + \underset{\sim}{\Omega}\times\underset{\sim}{r}_{A/B}$$

$$\underset{\sim}{a}_A = \underset{\sim}{a}_B + \underset{\sim}{a}_{Arel} + 2\left(\underset{\sim}{\omega}\times\underset{\sim}{v}_{Arel}\right) + \left(\underset{\sim}{\alpha}\times\underset{\sim}{r}_{A/B}\right) - \omega^2\underset{\sim}{r}_{A/B}$$

$$= \underset{\sim}{a}_B + (\underset{\sim}{a}_{A/B})_{xyz} + 2\left(\underset{\sim}{\Omega}\times(\underset{\sim}{v}_{A/B})_{xyz}\right) + \left(\dot{\underset{\sim}{\Omega}}\times\underset{\sim}{r}_{A/B}\right) - \Omega^2\underset{\sim}{r}_{A/B}$$