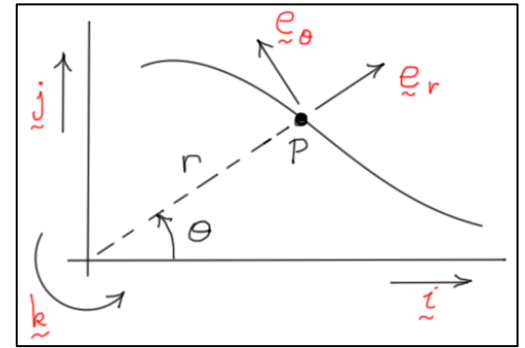


Intermediate Dynamics

Exercises #1

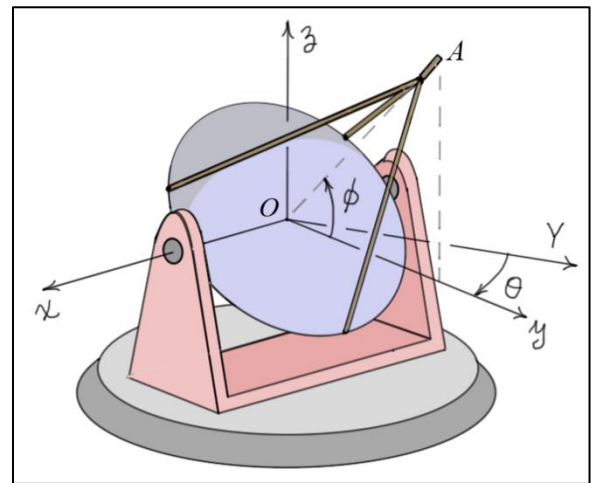
- 1) Radial and Transverse Components: The diagram shows two reference frames and a point P . The unit vector set $(\underline{i}, \underline{j}, \underline{k})$ defines the fixed frame R , and the unit vector set $(\underline{e}_r, \underline{e}_\theta, \underline{k})$ defines the rotating frame E . Given that the position vector of a point P is $\underline{r}_P = r \underline{e}_r$ and that ${}^R\omega_E = \dot{\theta} \underline{k}$, show that the velocity and acceleration of P can be written as



$$\underline{v}_P = \dot{r} \underline{e}_r + r \dot{\theta} \underline{e}_\theta$$

$$\underline{a}_P = (\ddot{r} - r \dot{\theta}^2) \underline{e}_r + (r \ddot{\theta} + 2\dot{r} \dot{\theta}) \underline{e}_\theta$$

- 2) The antenna system shown has two components, the base B and the antenna dish D . The distance from O to A is L . The axes (x, y, z) rotate with the base B . At any instant, the angle between the y -axis and the fixed Y -axis is given by the angle θ , and the angle between OA and the y -axis is given by the angle ϕ . Assuming you know values for θ , ϕ , and their first and second time derivatives, a) find ${}^R\omega_D$ and ${}^R\alpha_D$ the angular velocity and angular acceleration of the dish D in a fixed reference frame R ; and b) find ${}^R\underline{v}_A$ and ${}^R\underline{a}_A$ the velocity and acceleration of point A in a fixed frame R using *direct differentiation*. (The figure is based on a figure in Hibbeler, *Engineering Mechanics*, 1998)



- 3) The system shown has three components, a vertical column C , a horizontal arm M , and a disk D . The disk has radius r and rotates relative to the arm at a rate of ω_3 (rad/sec). The arm has length L and rotates relative to the column at a rate of ω_1 (rad/sec). The column rotates at a rate of ω_2 (rad/sec). Given that ω_1 , ω_2 , and ω_3 are *not* constant, a) find ${}^R\omega_D$ and ${}^R\alpha_D$ the angular velocity and angular acceleration of the disk D in the fixed reference frame R ; and b) find ${}^R\underline{v}_A$ and ${}^R\underline{a}_A$ the velocity and acceleration of point A in a fixed frame R using *direct differentiation*. Hint: You will find it necessary to define an angle ϕ between the plane of the disk and the $(\underline{e}_1, \underline{e}_2)$ plane.

