Intermediate Dynamics
Exercises #1 Answers

1. Answers given.

2. Reference frame $B:(\underset{\sim}{e}_1,\underset{\sim}{e}_2,\underset{\sim}{k})$ along the (x,y,z) axes.

   a) $^R\underset{\sim}{\omega}_D = \dot{\phi}\,\underset{\sim}{e}_1 - \dot{\theta}\,\underset{\sim}{k}$

   $^R\underset{\sim}{\alpha}_D = \ddot{\phi}\,\underset{\sim}{e}_1 - \dot{\theta}\dot{\phi}\,\underset{\sim}{e}_2 - \ddot{\theta}\,\underset{\sim}{k}$

   b) $\underset{\sim}{v}_A = L\left(\dot{\theta}C_\phi\,\underset{\sim}{e}_1 - \dot{\phi}S_\phi\,\underset{\sim}{e}_2 + \dot{\phi}C_\phi\,\underset{\sim}{k}\right)$

   $\underset{\sim}{a}_A = L\left(\left(\ddot{\theta}C_\phi - 2\dot{\theta}\dot{\phi}S_\phi\right)\underset{\sim}{e}_1 - \left(\ddot{\phi}S_\phi + \dot{\phi}^2C_\phi + \dot{\theta}^2C_\phi\right)\underset{\sim}{e}_2 + \left(\ddot{\phi}C_\phi - \dot{\phi}^2S_\phi\right)\underset{\sim}{k}\right)$

3. Reference frames: R (fixed frame), $C:(\underset{\sim}{e}_1,\underset{\sim}{e}_2,\underset{\sim}{e}_3)$, and $M:(\underset{\sim}{e}_1,\underset{\sim}{n}_2,\underset{\sim}{n}_3)$
   ($\underset{\sim}{n}_3$ is normal to the disk, $\underset{\sim}{n}_2 = \underset{\sim}{n}_3 \times \underset{\sim}{e}_1$, and ϕ is the angle between the disk and the $(\underset{\sim}{e}_1,\underset{\sim}{e}_2)$ plane.)

   a) $^R\underset{\sim}{\omega}_D = \omega_1\,\underset{\sim}{e}_1 + \omega_2 C_\phi\,\underset{\sim}{n}_2 + (\omega_3 - \omega_2 S_\phi)\underset{\sim}{n}_3$

   $^R\underset{\sim}{\alpha}_D = (\dot{\omega}_1 + \omega_2\omega_3 C_\phi)\underset{\sim}{e}_1 + (\dot{\omega}_2 C_\phi - \omega_1\omega_2 S_\phi - \omega_1\omega_3)\underset{\sim}{n}_2 +$
   $(\dot{\omega}_3 - \dot{\omega}_2 S_\phi - \omega_1\omega_2 C_\phi)\underset{\sim}{n}_3$

   b) $^R\underset{\sim}{v}_A = r(\omega_2 S_\theta S_\phi - \omega_3 S_\theta)\underset{\sim}{e}_1 + (r\,\omega_3 C_\theta - (L + r\,C_\theta)\omega_2 S_\phi)\underset{\sim}{n}_2 +$
   $(r\,\omega_1 S_\theta - (L + r\,C_\theta)\omega_2 C_\phi)\underset{\sim}{n}_3$

   $^R\underset{\sim}{a}_A = (r\,\dot{\omega}_2 S_\theta S_\phi - r\,\dot{\omega}_3 S_\theta - r\,\omega_3^2 C_\theta - (L + r\,C_\theta)\omega_2^2 + 2r\,\omega_1\omega_2 S_\theta C_\phi + 2r\,\omega_2\omega_3 S_\phi C_\theta)\underset{\sim}{e}_1 +$
   $(r\,\dot{\omega}_3 C_\theta - (L + r\,C_\theta)\dot{\omega}_2 S_\phi - r\,\omega_1^2 S_\theta - r\,\omega_2^2 S_\theta S_\phi^2 - r\,\omega_3^2 S_\theta + 2r\,\omega_2\omega_3 S_\theta S_\phi)\underset{\sim}{n}_2 +$
   $(r\,\dot{\omega}_1 S_\theta - (L + r\,C_\theta)\dot{\omega}_2 C_\phi - r\,\omega_2^2 S_\theta S_\phi C_\phi + 2r\,\omega_2\omega_3 S_\theta C_\phi + 2r\,\omega_1\omega_3 C_\theta)\underset{\sim}{n}_3$