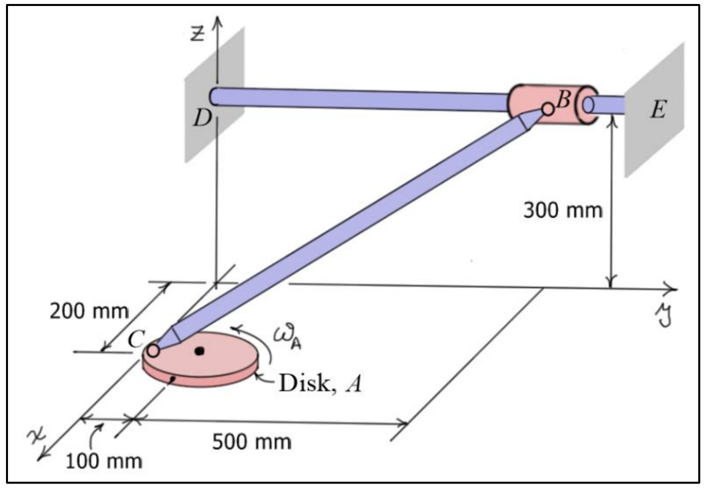
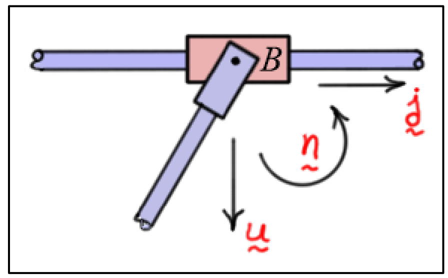
Intermediate Dynamics
Exercises #3

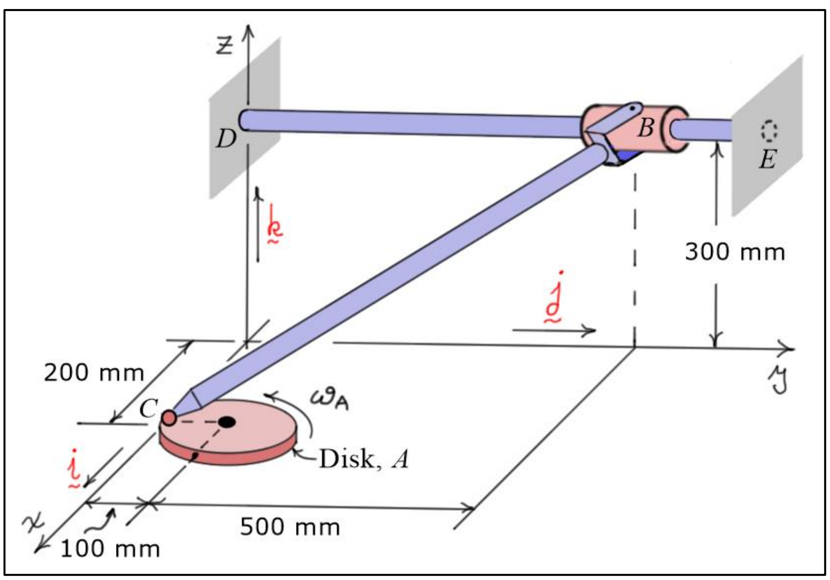
1) The figure shows a three-dimensional slider crank mechanism. The (x, y, z) axes shown represent a fixed reference frame R. At the instant shown, disk A has angular velocity ${}^{R}\underset{\sim}{\omega}_{A} = 10\underset{\sim}{k}$ (rad/sec).



a) If BC is attached to both the disk and the collar with ball-and-socket joints, find ${}^{R}\underset{\sim}{\omega}_{BC}$ and ${}^{R}\underset{\sim}{v}_{B}$. In this case, assume that ${}^{R}\underset{\sim}{\omega}_{BC}$ is ***perpendicular*** to BC. (Hibbeler, 1995)

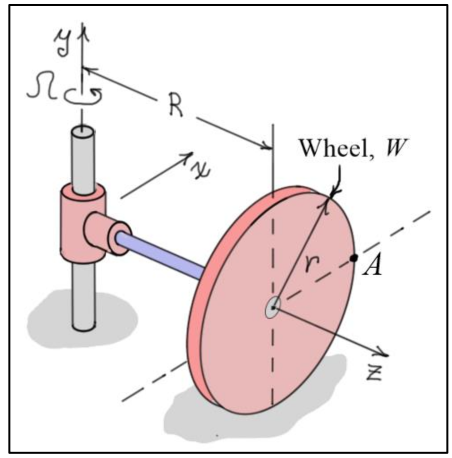
b) If the ball-and-socket joint at the collar is replaced with the ***two-axis joint*** shown at the right, find ${}^{R}\underset{\sim}{\omega}_{BC}$ and ${}^{R}\underset{\sim}{v}_{B}$. In this case, the rod BC rotates relative to collar C about the axis defined by the unit vector $\underset{\sim}{n}$ which is perpendicular to the plane BCD, and the collar translates and rotates about the $\underset{\sim}{j}$ direction. In this case, ${}^{R}\underset{\sim}{\omega}_{BC}$ is ***perpendicular*** to the vector $\underset{\sim}{u} = \underset{\sim}{j} \times \underset{\sim}{n}$. (Hibbeler, 1995)



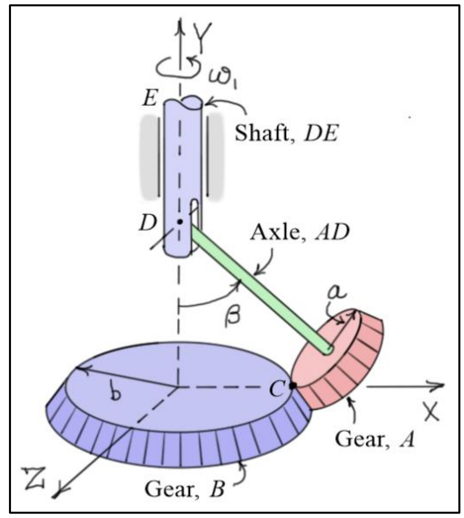


Slider-Crank with Joint of Part (b)

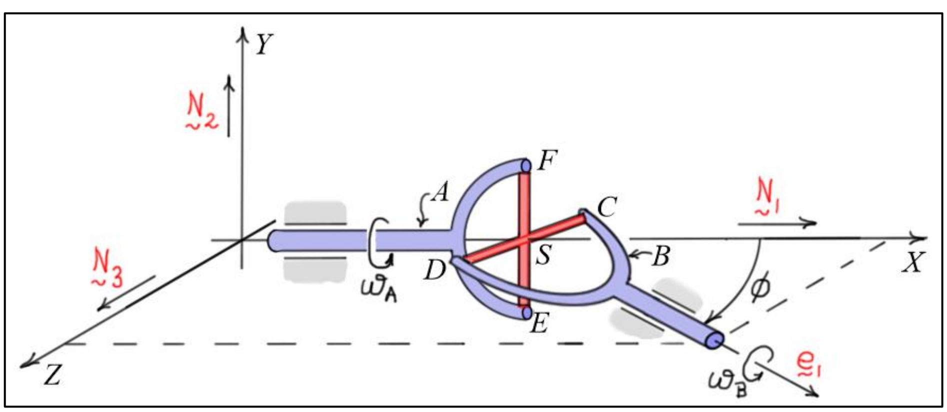
2) The system shown consists of a vertical column, a horizontal axle, and a wheel of radius r. The horizontal arm rotates at a ***constant*** rate Ω, and the wheel (*W*) rolls without slipping in a circular arc. Find $\underset{\sim}{\omega}_W$ and $\underset{\sim}{\alpha}_W$ the angular velocity and angular acceleration of the wheel relative to a fixed frame, and find $\underset{\sim}{v}_A$ and $\underset{\sim}{a}_A$ the velocity and acceleration of point *A*. (Meriam and Kraige, 1992)



3) In the system shown, beveled gear *A* rolls on beveled gear *B*. As it rolls on *B* it spins about the axle *AD* which is pinned to the vertical shaft *DE*. If *DE* rotates at a ***constant*** angular velocity ω_1 (rad/sec), find ${}^{AD}\underset{\sim}{\omega}_A$ the angular velocity of gear *A* relative to the axle *AD*, ${}^{R}\underset{\sim}{\omega}_A$ the angular velocity of gear *A*, ${}^{R}\underset{\sim}{\alpha}_A$ the angular acceleration of gear *A*, and $\underset{\sim}{a}_C$ the acceleration of tooth *C* of gear *A*. (Beer & Johnston, 1977)



4) The diagram below depicts a yoke-and-spider ***universal joint***. The joint has three members – the ***input shaft*** *A*, ***output shaft*** *B*, and ***spider*** *S*. Shaft *A* rotates about the *X*-axis, and shaft *B* rotates about the $\underset{\sim}{e}_1$ direction which is in the *XZ* plane and makes an angle of ϕ with the *X*-axis. ***At the instant shown***, spider *S* rotates ***relative*** to shaft *A* about the direction of spider segment *EF* (represented by unit vector $\underset{\sim}{n}_{EF} = \underset{\sim}{N}_2$), and it rotates ***relative*** to shaft *B* about the spider segment *CD* (represented by unit vector $\underset{\sim}{n}_{CD} = \underset{\sim}{e}_1 \times \underset{\sim}{n}_{EF} = \underset{\sim}{e}_1 \times \underset{\sim}{N}_2$). ***At the instant shown***, find ω_B / ω_A the ratio of the speed of shaft *B* to the speed of shaft *A*. Hint: Apply the ***summation rule*** for angular velocities through the joint using the known directions for the angular velocities and relative angular velocities.



5) Referring again to the diagram of the yoke-and-spider ***universal joint*** of problem (4), let input shaft A rotate through an angle θ. The spider segment EF is now at an angle θ relative to the Y-axis, so $\underset{\sim}{n}_{EF} = C_\theta \underset{\sim}{N}_2 + S_\theta \underset{\sim}{N}_3$. As before, shaft B rotates about the $\underset{\sim}{e}_1$ direction, and $\underset{\sim}{n}_{CD} = \underset{\sim}{e}_1 \times \underset{\sim}{n}_{EF}$. Find ω_B / ω_A the ratio of the speed of shaft B to the speed of shaft A as a function of the input shaft angle θ. Hint: Apply the ***summation rule*** for angular velocities ***through the joint*** using the known directions for the angular velocities and relative angular velocities.