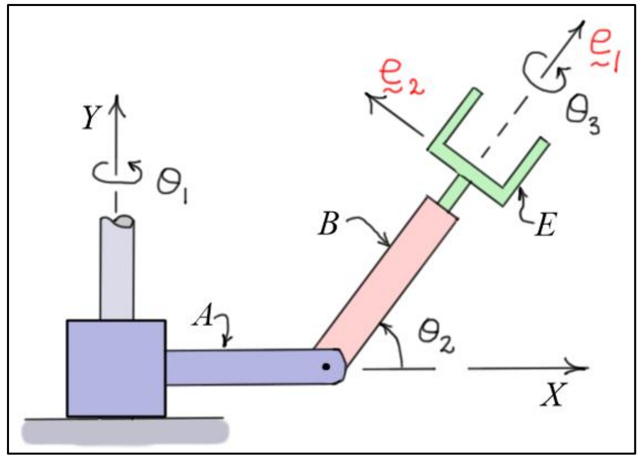
Intermediate Dynamics
Exercises #5

1. The system shown consists of three components, the arms A and B and the end-effector E. The orientation of E relative to a fixed frame is described by the three angles shown. Note that the sequence of rotations θ_1, θ_2, and θ_3 is a 2-3-1 body-fixed rotation sequence. Complete the following: a) Derive the transformation matrix $[R]$ that relates the unit vectors $(\underset{\sim}{e}_1, \underset{\sim}{e}_2, \underset{\sim}{e}_3)$ (fixed in E) to the unit vectors $(\underset{\sim}{N}_1, \underset{\sim}{N}_2, \underset{\sim}{N}_3)$ of the fixed frame.



b) Find the $\underset{\sim}{e}_i$ components of ${}^R\underset{\sim}{\omega}_E$ the angular velocity of E in R. c) Invert the equations from part (b) to solve for $\dot{\theta}_1$, $\dot{\theta}_2$, and $\dot{\theta}_3$ in terms of the angular velocity components. d) Find ${}^R\underset{\sim}{\alpha}_E$ the angular acceleration of E relative to the fixed frame. Note: In all parts of this problem, assume the angles θ_1, θ_2, θ_3 and their time-derivatives are all ***nonzero***.

2. For the yoke-and-spider ***universal joint*** shown below, the unit vectors fixed in the shaft $B:\left(\underset{\sim}{e}_1, \underset{\sim}{e}_2, \underset{\sim}{e}_3\right)$ are oriented relative to the fixed-frame $R:\left(\underset{\sim}{N}_1, \underset{\sim}{N}_2, \underset{\sim}{N}_3\right)$ using a 1-2-3 body-fixed rotation sequence. The figure shows the configuration where all the angles are ***zero***. In its final configuration, the shaft B is aligned with the unit vector $\underset{\sim}{n}$ so that $\underset{\sim}{e}_1 = C_\phi \underset{\sim}{N}_1 + S_\phi \underset{\sim}{N}_3$. Using results in the notes for a 1-2-3 body-fixed rotation sequence, complete the following: a) Show that $C_2 C_3 = C_\phi$ and $S_3 = S_1 S_\phi$. b) Show that $\omega_B = \left(C_\phi \big/ \left(1 - S_1^2 S_\phi^2\right)\right)\dot{\theta}_1$ where ω_B is the angular speed of shaft B about the $\underset{\sim}{n}$ direction. c) Plot the ratio $\left(\omega_B / \dot{\theta}_1\right)$ versus θ_1 for one complete revolution of shaft A for $\phi = 20^{\circ}$ and $\phi = 40^{o}$.