

## Intermediate Dynamics

### Exercises #5 Answers

$$1. (a) [R] = \begin{bmatrix} C_1 C_2 & S_2 & -S_1 C_2 \\ S_1 S_3 - C_1 S_2 C_3 & C_2 C_3 & C_1 S_3 + S_1 S_2 C_3 \\ S_1 C_3 + C_1 S_2 S_3 & -C_2 S_3 & C_1 C_3 - S_1 S_2 S_3 \end{bmatrix}$$

$$(b) {}^R \omega_E = (\dot{\theta}_3 + \dot{\theta}_1 S_2) \underline{e}_1 + (\dot{\theta}_2 S_3 + \dot{\theta}_1 C_2 C_3) \underline{e}_2 + (\dot{\theta}_2 C_3 - \dot{\theta}_1 C_2 S_3) \underline{e}_3$$

$$(c) \dot{\theta}_1 = (\omega_2 C_3 - \omega_3 S_3) / C_2$$

$$\dot{\theta}_2 = \omega_2 S_3 + \omega_3 C_3$$

$$\dot{\theta}_3 = \omega_1 + \left[ S_2 (\omega_3 S_3 - \omega_2 C_3) / C_2 \right]$$

$$d) {}^R \alpha_E = \frac{{}^R d}{{}^R dt} ({}^R \omega_E) = \frac{{}^E d}{{}^E dt} ({}^R \omega_E) + ({}^R \omega_E \times {}^R \omega_E) = \frac{{}^E d}{{}^E dt} ({}^R \omega_E)$$

$$\Rightarrow \boxed{{}^R \alpha_E = \dot{\omega}_1 \underline{e}_1 + \dot{\omega}_2 \underline{e}_2 + \dot{\omega}_3 \underline{e}_3}$$

Here,

$$\dot{\omega}_1 = \ddot{\theta}_1 S_2 + \dot{\theta}_1 \dot{\theta}_2 C_2 + \ddot{\theta}_3$$

$$\dot{\omega}_2 = \ddot{\theta}_1 C_2 C_3 - \dot{\theta}_1 \dot{\theta}_2 S_2 C_3 - \dot{\theta}_1 \dot{\theta}_3 C_2 S_3 + \dot{\theta}_2 \dot{\theta}_3 C_3 + \ddot{\theta}_2 S_3$$

$$\dot{\omega}_3 = -\ddot{\theta}_1 C_2 S_3 + \dot{\theta}_1 \dot{\theta}_2 S_2 S_3 - \dot{\theta}_1 \dot{\theta}_3 C_2 C_3 - \dot{\theta}_2 \dot{\theta}_3 S_3 + \ddot{\theta}_2 C_3$$

2. (a)-(b) Results given in problem definition.