

## Intermediate Dynamics

### Exercises #10 Answers

1.  $\theta_{eq} = 0$  and  $\theta_{eq} = 45$  (deg)

$$2. \begin{bmatrix} (m_1 + m_2) & \frac{1}{2} m_2 \ell \\ \frac{1}{2} m_2 \ell & \frac{1}{3} m_2 \ell^2 \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} c & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} k & 0 \\ 0 & \frac{1}{2} m_2 g \ell \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} F(t) \\ 0 \end{Bmatrix}$$

$$\{\omega_n\} \approx \begin{Bmatrix} 4.89 \\ 29.44 \end{Bmatrix} \text{ (r/s)} \approx \begin{Bmatrix} 0.7784 \\ 4.686 \end{Bmatrix} \text{ (Hz)}$$

Mode 1:  $\underline{u}_1 \approx \begin{Bmatrix} 1 \\ 77.8 \end{Bmatrix}$

Mode 2:  $\underline{u}_2 \approx \begin{Bmatrix} 1 \\ -0.7715 \end{Bmatrix}$

3.  $\theta_{eq} = 0$  and  $\theta_{eq} = 90^0$

$$\theta_{eq} = 0 \Rightarrow \Delta \ddot{\theta} - \Omega^2 (\Delta \theta) = 0 \quad \text{(unstable equilibrium)}$$

$$\theta_{eq} = 90^0 \Rightarrow \Delta \ddot{\theta} + \Omega^2 (\Delta \theta) = 0 \quad \text{(stable equilibrium)}$$

$$M_\phi = \frac{1}{6} m \ell^2 S_\theta C_\theta \Omega \dot{\theta}$$

4.  $\theta_{eq} = 58.7^0 \Rightarrow \Delta \ddot{\theta} + (133.52) \Delta \theta = 0$  (stable equilibrium)

$$M_\phi = \frac{2}{3} m \ell^2 S_\theta C_\theta \Omega \dot{\theta}$$