Intermediate Dynamics
Exercises #11 Answers

1. Lagrangian: $L = \frac{1}{8} m\left(\dot{x}^2 + \dot{y}^2\right) + \frac{1}{2} m_p \dot{y}^2 + \frac{1}{24} m \ell^2 \dot{\theta}^2 - m_p g\, y - \frac{1}{2} m g\, y - \frac{1}{2} k\, x^2$

   Three differential/algebraic equations from Lagrange's equations:

$$\begin{aligned} \tfrac{1}{4} m\ddot{x} + c\dot{x} + kx &= \lambda_1 \\ (m_p + \tfrac{1}{4} m)\ddot{y} + (m_p + \tfrac{1}{2} m)g &= \lambda_2 - F(t) \\ \left(\tfrac{1}{12} m\ell^2\right)\ddot{\theta} &= \left(-\ell C_\theta\right)\lambda_1 + \left(\ell S_\theta\right)\lambda_2 \end{aligned}$$

   Two differentiated constraint equations:

$$\begin{aligned} \ddot{x} - \left(\ell C_\theta\right)\ddot{\theta} + \left(\ell S_\theta\right)\dot{\theta}^2 &= 0 \\ \ddot{y} + \left(\ell S_\theta\right)\ddot{\theta} + \left(\ell C_\theta\right)\dot{\theta}^2 &= 0 \end{aligned}$$

2. Lagrangian: $L = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 \left[(\dot{x} + \dot{x}_G)^2 + \dot{y}_G^2\right] + \frac{1}{24} m_2 \ell^2 \dot{\theta}^2 - \frac{1}{2} k x^2 + m_2 g y_G$

   Four differential/algebraic equations from Lagrange's equations:

$$\begin{aligned} (m_1 + m_2)\ddot{x} + m_2 \ddot{x}_G + c\dot{x} + kx &= F(t) \\ m_2 \ddot{x} + m_2 \ddot{x}_G &= \lambda_1 \\ m_2 \ddot{y}_G - m_2 g &= \lambda_2 \\ \left(\tfrac{1}{12} m_2 \ell^2\right)\ddot{\theta} &= \left(-\tfrac{1}{2}\ell C_\theta\right)\lambda_1 + \left(\tfrac{1}{2}\ell S_\theta\right)\lambda_2 \end{aligned}$$

   Two differentiated constraint equations:

$$\begin{aligned} \ddot{x}_G - \left(\tfrac{1}{2}\ell C_\theta\right)\ddot{\theta} + \left(\tfrac{1}{2}\ell S_\theta\right)\dot{\theta}^2 &= 0 \\ \ddot{y}_G + \left(\tfrac{1}{2}\ell S_\theta\right)\ddot{\theta} + \left(\tfrac{1}{2}\ell C_\theta\right)\dot{\theta}^2 &= 0 \end{aligned}$$