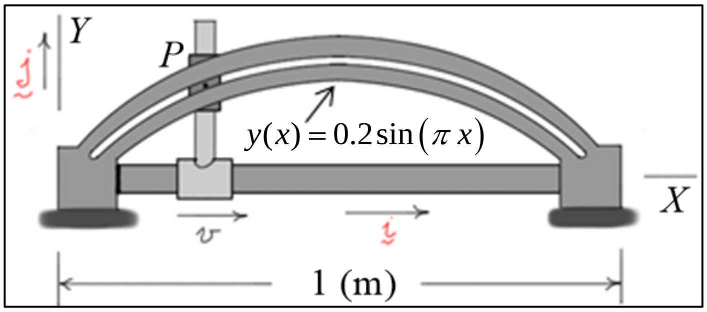
Elementary Dynamics Example #4: (Two-Dimensional Motion, Rectangular Components)



Given: $v_x = \dot{x} = 2$ (m/s) … constant

Find: $|\underset{\sim}{v}_P|$ and $|\underset{\sim}{a}_P|$ when $x = 0.25$ (m)

Solution:

To find $\underset{\sim}{v}_P$ we can ***differentiate*** the ***position vector*** of P relative to the origin.

$$\boxed{\underset{\sim}{r}_P = x\,\underset{\sim}{i} + y\,\underset{\sim}{j}} \Rightarrow \boxed{\underset{\sim}{v}_P = d\underset{\sim}{r}_P/dt = \dot{x}\,\underset{\sim}{i} + \dot{y}\,\underset{\sim}{j}} \Rightarrow \begin{cases} \dot{x} = v_x = 2 \text{ (m/s)} \\ \dot{y} = \dfrac{d}{dt}\big(0.2\sin(\pi x)\big) = 0.2\cos(\pi x)(\pi\dot{x}) \end{cases}$$

At $x = 0.25$ (m), $\boxed{\dot{y} = 0.2(2\pi)\cos\left(\frac{\pi}{4}\right) = 0.2\pi\sqrt{2} \approx 0.889 \text{ (m/s)}}$

$$\Rightarrow \boxed{\underset{\sim}{v}_P = 2\,\underset{\sim}{i} + 0.889\,\underset{\sim}{j}} \Rightarrow \boxed{|\underset{\sim}{v}_P| = \sqrt{2^2 + \left(0.2\pi\sqrt{2}\right)^2} \approx 2.19 \text{ (m/s)}}$$

To find $\underset{\sim}{a}_P$ we can ***differentiate*** the ***velocity vector***.

$$\boxed{\underset{\sim}{v}_P = d\underset{\sim}{r}_P/dt = \dot{x}\,\underset{\sim}{i} + \dot{y}\,\underset{\sim}{j}} \Rightarrow \boxed{\underset{\sim}{a}_P = d\underset{\sim}{v}_P/dt = \ddot{x}\,\underset{\sim}{i} + \ddot{y}\,\underset{\sim}{j}}$$

where $\boxed{\ddot{x} = 0}$ and $\boxed{\ddot{y} = \dfrac{d}{dt}\big(0.2\pi\dot{x}\cos(\pi x)\big) = \underbrace{0.2\pi\ddot{x}\cos(\pi x)}_{\text{zero}} - 0.2(\pi\dot{x})^2\sin(\pi x)}$

At $x = 0.25$ (m), $\boxed{\ddot{y} = -0.2(\pi\dot{x})^2\sin(\pi x) = -0.2(2\pi)^2\sin\left(\frac{\pi}{4}\right) \approx 5.58 \text{ (m/s}^2)}$

$$\Rightarrow \boxed{|\underset{\sim}{a}_P| \approx 5.58 \text{ (m/s}^2)}$$