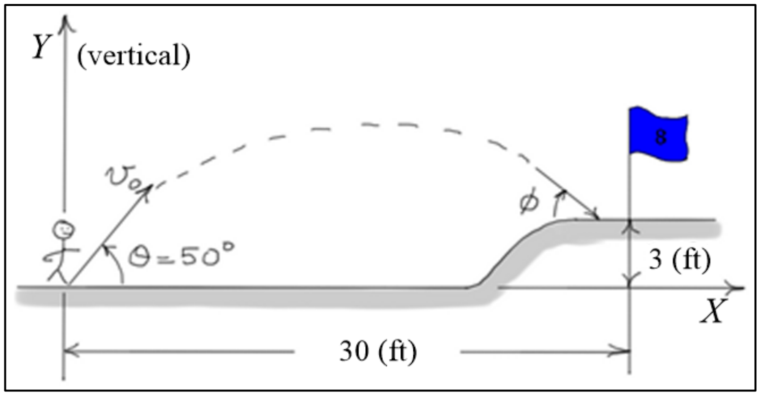
Elementary Dynamics Example #5: (Rectangular Components, Projectile Motion)

Given: neglect air resistance
initial velocity v_0 at $\theta = 50$ (deg)
ball lands 3 feet short of the hole
at $(27,3)$ (ft)

Find: v_0 and angle ϕ at which the ball strikes the ground



Solution:

X direction: $x(t) = x_0 + v_{x_0} t = (v_0 \cos(50))t$ — constant velocity in X direction

Y direction: $y(t) = y_0 + v_{y_0} t - \frac{1}{2} g t^2 = (v_0 \sin(50))t - \frac{1}{2} g t^2$ — constant acceleration in Y direction

When the ***ball strikes the ground***:

$$x = 27 = v_0 \cos(50) t \quad \Rightarrow \quad t = 27/(v_0 \cos(50))$$

$$y = 3 = (v_0 \sin(50))t - \tfrac{1}{2} g t^2 = (\not{v}_0 \sin(50)) 27/(\not{v}_0 \cos(50)) - \tfrac{1}{2} g \left(27/(v_0 \cos(50))\right)^2$$

$$= 27 \tan(50) - \frac{27^2 g}{2 v_0^2 \cos^2(50)} \quad \Rightarrow \quad \frac{27^2 g}{2 v_0^2 \cos^2(50)} = 27 \tan(50) - 3$$

$$\Rightarrow \quad v_0 = \sqrt{\frac{27^2 g}{2\cos^2(50)(27\tan(50) - 3)}} \approx 31.2 \text{ (ft/s)} \quad \ldots \text{ initial velocity}$$

At what angle does the ball strike the ground? The ***velocity vector*** is ***tangent*** to the path, so we can use the X and Y components of the velocity of the ball when it strikes the ground.

$$v_x = v_{x_0} = v_0 \cos(50) \approx 20.056 \text{ (ft/s)} \quad \text{(constant)}$$

$$v_y^2 = v_{y_0}^2 - 2g\Delta y = (v_0 \sin(50))^2 - 2g(3) \quad \Rightarrow \quad v_y = \sqrt{(v_0 \sin(50))^2 - 2g(3)} \approx 19.445 \text{ (ft/s)}$$

$$\phi = \tan^{-1}\left(\frac{v_y}{v_x}\right) \approx 44.1 \text{ (deg)}$$