

Elementary Dynamics Example #9: (2D Motion, Radial & Transverse Components)

Given: $r = 5 \text{ (mi)} = 26,400 \text{ (ft)}$

$\dot{r} = 1 \text{ (mi/s)} = 5280 \text{ (ft/s)}$

$\ddot{r} = 0.01 \text{ (mi/s}^2\text{)} = 52.8 \text{ (ft/s}^2\text{)}$

$\dot{\theta} = 0.1 \text{ (deg/s)} = 0.1\pi/180 \text{ (rad/s)}$

$\ddot{\theta} = -0.05 \text{ (deg/s}^2\text{)} = -0.05\pi/180 \text{ (rad/s}^2\text{)}$

Find: \underline{v} and \underline{a} in ft/s and ft/s² using *radial* and *transverse* components.

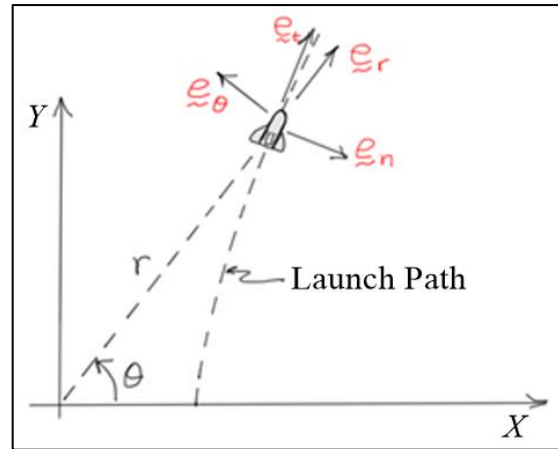
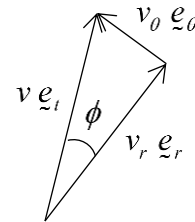
Solution:

Velocity:

$$\underline{v} = \dot{r} \underline{e}_r + r \dot{\theta} \underline{e}_\theta = 5280 \underline{e}_r + 26400(0.1\pi/180) \underline{e}_\theta \approx 5280 \underline{e}_r + 46.1 \underline{e}_\theta \text{ (ft/s)}$$

$$|\underline{v}| = \sqrt{v_r^2 + v_\theta^2} \approx 5280 \text{ (ft/s)} \text{ (tangent to the launch path)}$$

$$\phi = \tan^{-1}\left(\frac{v_\theta}{v_r}\right) = \tan^{-1}\left(\frac{46.1}{5280}\right) \approx 0.5 \text{ (deg)}$$



Acceleration:

$$\underline{a} = (\ddot{r} - r \dot{\theta}^2) \underline{e}_r + (r \ddot{\theta} + 2\dot{r} \dot{\theta}) \underline{e}_\theta$$

$$a_r = \ddot{r} - r \dot{\theta}^2 = 52.8 - 26400(0.1\pi/180)^2 \approx 52.7196 \approx 52.7 \text{ (ft/s}^2\text{)}$$

$$a_\theta = r \ddot{\theta} + 2\dot{r} \dot{\theta} = 26400(-0.05\pi/180) + 2(5280)(0.1\pi/180) \approx -4.6076 \approx -4.61 \text{ (ft/s}^2\text{)}$$

$$a = \sqrt{a_r^2 + a_\theta^2} \approx 52.9206 \approx 52.9 \text{ (ft/s}^2\text{)} \approx 1.64 \text{ g's}$$