

Elementary Dynamics Example #10: (2D Motion, Radial & Transverse Components)

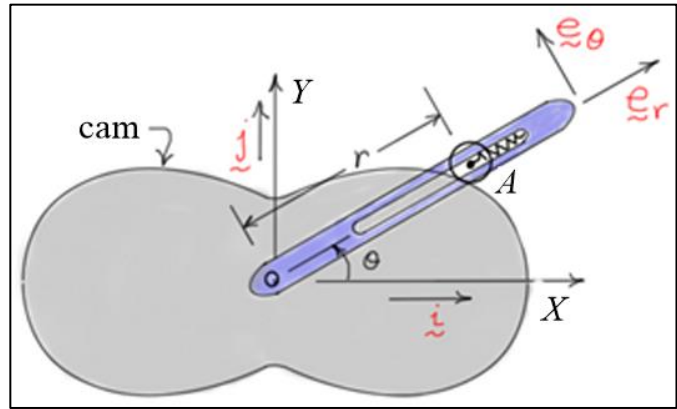
Given: $\dot{\theta} = \omega = 10$ (rad/s) ... constant

$$r(\theta) = 1 + 0.5\cos(2\theta) \text{ (ft)}$$

Find: v_A and a_A in ft/s and ft/s² using **radial** and **transverse** components at $\theta = 30$ (deg) .

Solution:

Velocity:



$$\underline{v} = v_r \underline{e}_r + v_\theta \underline{e}_\theta$$

$$v_r = \frac{dr}{dt} = \frac{d}{dt}(1 + 0.5\cos(2\theta)) = 0.5(-\sin(2\theta))(2\dot{\theta})$$

$$= -\dot{\theta}\sin(2\theta) \approx -8.66025 \approx -8.66 \text{ (ft/s)}$$

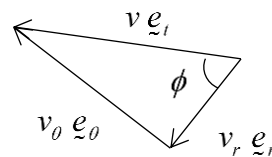
chain rule

$$v_\theta = r\dot{\theta} = (1 + 0.5\cos(2\theta))(10) = (1.25)(10) = 12.5 \text{ (ft/s)}$$

$$\underline{v}_A = -8.66 \underline{e}_r + 12.5 \underline{e}_\theta \text{ (ft/s)}$$

$$|\underline{v}_A| = \sqrt{v_r^2 + v_\theta^2} \approx 15.2069 \approx 15.2 \text{ (ft/s)} \text{ (tangent to the cam surface)}$$

$$\phi = \tan^{-1}\left(\frac{v_\theta}{|v_r|}\right) = \tan^{-1}\left(\frac{12.5}{8.66}\right) \approx 55.3 \text{ (deg)}$$



Acceleration:

$$\underline{a} = (\ddot{r} - r\dot{\theta}^2) \underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta}) \underline{e}_\theta$$

$$\ddot{r} = \frac{d\dot{r}}{dt} = \frac{d}{dt}(-\dot{\theta}\sin(2\theta)) = \underbrace{-\ddot{\theta}\sin(2\theta)}_{\text{zero}} - \dot{\theta}(\cos(2\theta))(2\dot{\theta}) = -2\dot{\theta}^2 \cos(2\theta)$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2\dot{\theta}^2 \cos(2\theta) - 1.25(10^2) = -225 \text{ (ft/s}^2\text{)}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + (2(-8.66025)(10)) = -173.205 \approx -173 \text{ (ft/s}^2\text{)}$$

$$\underline{a}_A = -225 \underline{e}_r - 173 \underline{e}_\theta \text{ (ft/s}^2\text{)}$$