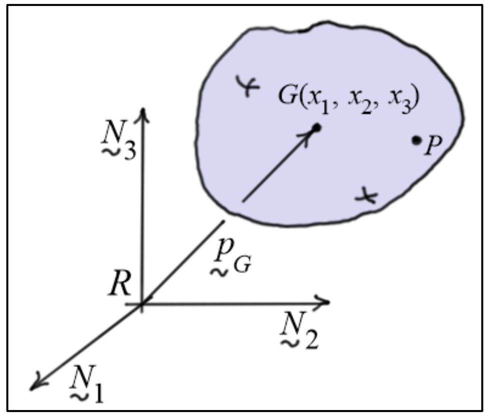
Multibody Dynamics
Velocities and Partial Velocities Using Absolute Coordinates

Mass Center Velocities and Partial Velocities



Consider the rigid body shown in the diagram. Given that (x_1, x_2, x_3) are the position coordinates of the mass center G relative to the fixed frame, then $\underset{\sim}{p}_G$ the position vector of G, ${}^R\underset{\sim}{v}_G$ the velocity of G in R, and $\left[v_{G,\dot{x}}\right]$ the partial velocity matrix for G may be written in matrix form as

$$\{p_G\} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}, \quad \{v_G\} = \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix}, \quad \left[v_{G,\dot{x}}\right] = \begin{bmatrix} \dfrac{\partial\, {}^R\underset{\sim}{v}_G}{\partial \dot{x}_1} & \dfrac{\partial\, {}^R\underset{\sim}{v}_G}{\partial \dot{x}_2} & \dfrac{\partial\, {}^R\underset{\sim}{v}_G}{\partial \dot{x}_3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Velocities of Other Points

For other points in the system, such as point P in the diagram above, the position vector can be written in terms of the mass-center position vector as follows

$$\{p_P\} = \{p_G\} + \{p_{P/G}\} = \{p_G\} + [R]^T \{p'_{P/G}\}$$

Differentiating this expression gives the velocity of P.

$$\{v_P\} = \{\dot{p}_P\} = \{\dot{p}_G\} + \left[\dot{R}\right]^T \{p'_{P/G}\} + [R]^T \underbrace{\{\dot{p}'_{P/G}\}}_{\text{zero}} = \{v_G\} + \left[\dot{R}\right]^T \{p'_{P/G}\} \qquad (1)$$

where $\{\dot{p}'_{P/G}\} = \{0\}$ because $\{p'_{P/G}\}$ represents the coordinates of P relative to G in the body frame in which are all constant.

To reduce Eq. (1) further, a decision must be made whether to use fixed-frame or body-frame components for the angular velocity vectors. The two results are

$$\boxed{\{v_P\} = \{v_G\} + [\tilde{\omega}_B][R]^T \{p'_{P/G}\}} \quad \text{or} \quad \boxed{\{v_P\} = \{v_G\} + [R]^T [\tilde{\omega}'_B]\{p'_{P/G}\}}$$

Note: As before, the “primes” indicate components in the body frame, and “no primes” indicate components in the fixed frame.