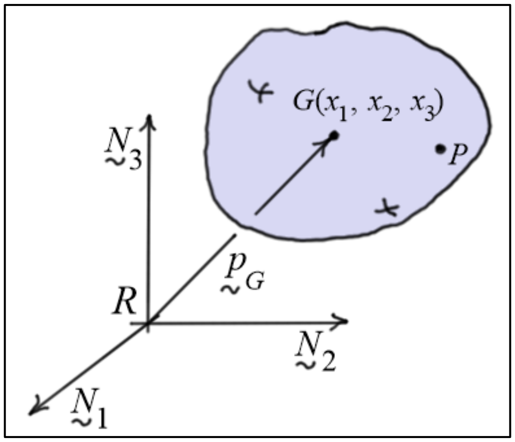
Multibody Dynamics
Accelerations Using Absolute Coordinates

Mass Center Accelerations



Consider the ***rigid body*** in the diagram. Given that (x_1, x_2, x_3) are the position coordinates of the mass center G relative to the fixed frame, then $\underset{\sim}{p}_G$ the position vector of G, ${}^R\underset{\sim}{v}_G$ the velocity of G, and ${}^R\underset{\sim}{a}_G$ the acceleration of G in R may be written in matrix form as

$$\{p_G\} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} \qquad \{v_G\} = \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} \qquad \{a_G\} = \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix}$$

Accelerations of Other Points

We can write the inertial components of the velocities of other points in the system, such as P in the diagram above, in terms of the inertial components of $\underset{\sim}{v}_G$ the velocity of G as follows:

$$\{v_P\} = \{v_G\} + [\tilde{\omega}_B][R]^T \{p'_{P/G}\}$$
$$\{v_P\} = \{v_G\} + [R]^T [\tilde{\omega}'_B]\{p'_{P/G}\}$$

The first equation is written in terms of the fixed-frame components of ${}^R\underset{\sim}{\omega}_B$ and the second is written in terms of the body-fixed components of ${}^R\underset{\sim}{\omega}_B$. These two equations can be ***differentiated*** to give the fixed-frame components of ${}^R\underset{\sim}{a}_P$ the acceleration of P.

$$\begin{aligned} \{a_P\} &= \{a_G\} + \left[\dot{\tilde{\omega}}_B\right][R]^T \{p'_{P/G}\} + [\tilde{\omega}_B]\left[\dot{R}\right]^T \{p'_{P/G}\} \\ &= \{a_G\} + [\tilde{\alpha}_B][R]^T \{p'_{P/G}\} + [\tilde{\omega}_B][\tilde{\omega}_B][R]^T \{p'_{P/G}\} \end{aligned} \quad (1)$$

$$\begin{aligned} \{a_P\} &= \{a_G\} + [R]^T \left[\dot{\tilde{\omega}}'_B\right]\{p'_{P/G}\} + \left[\dot{R}\right]^T [\tilde{\omega}'_B]\{p'_{P/G}\} \\ &= \{a_G\} + [R]^T [\tilde{\alpha}'_B]\{p'_{P/G}\} + [R]^T [\tilde{\omega}'_B][\tilde{\omega}'_B]\{p'_{P/G}\} \end{aligned} \quad (2)$$