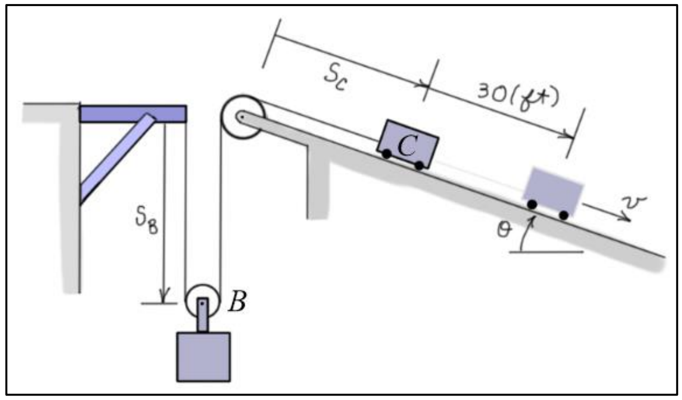
Elementary Dynamics Example #22: (Conservation of Energy)



Given: $W_C = 600$ (lb), $W_B = 200$ (lb)
$\theta = 20$ (deg)
system is released from ***rest***
cart travels ***down*** the incline

Find: v_C when the cart is 30 (ft) down the incline

Solution: Conservation of energy for the system

Kinematics:

$$\boxed{2s_B + s_C = \text{constant}} \quad \Rightarrow \quad \boxed{v_C = 2v_B}$$

Conservation of Energy: (choosing the datum for each mass at the starting positions)

$$\underbrace{K_1}_{\text{zero (at rest)}} + \underbrace{V_1}_{\text{zero (each mass at its datum)}} = K_2 + V_2$$

$$K_2 = \tfrac{1}{2}\left(\frac{W_C}{g}\right)v_C^2 + \tfrac{1}{2}\left(\frac{W_B}{g}\right)\left(\tfrac{1}{2}v_C\right)^2 = \frac{1}{2g}\left(W_C + \tfrac{1}{4}W_B\right)v_C^2 = \frac{325}{g}v_C^2$$

$$V_2 = W_B(15) - W_C\left(30\sin(20)\right) = -3156.36 \approx -3156 \text{ (ft-lb)}$$

Substituting into the conservation of energy equation:

$$\left(\tfrac{325}{g}\right)v_C^2 = 3156.36 \quad \Rightarrow \quad \boxed{v_C = \sqrt{\frac{3156.36\,g}{325}} = 17.684 \approx 17.7 \text{ (ft/s)}}$$

Note: Tension "*T*" does ***negative*** work on *C*, and tension "2*T*" does ***positive*** work on *B*. But because *B* moves just ***half*** the distance that *C* moves, the net work of the tension *T* on the system is ***zero***.