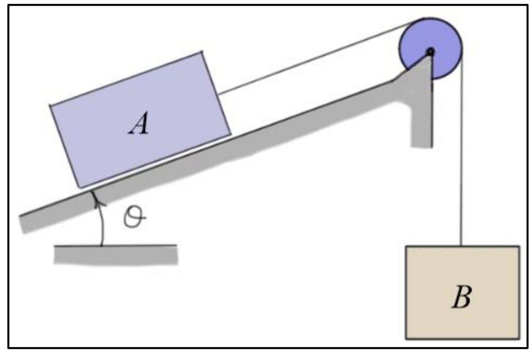
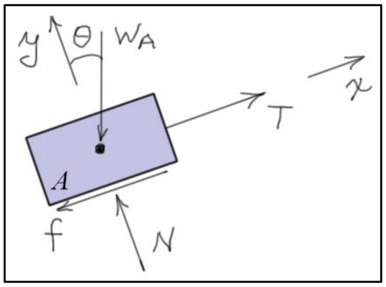
Elementary Dynamics Example #23: (Impulse & Momentum)

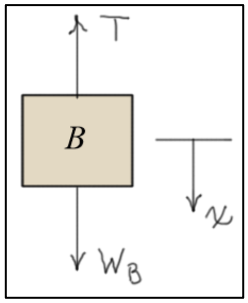
Given: $W_A = 100$ (lb), $W_B = 110$ (lb), $\mu_k = 0.3$
$\theta = 20$ (deg), system is ***released from rest***
A moves ***up the plane*** and B moves ***down***

Find: v_2 the velocities of the two blocks $\frac{1}{2}$ (sec) after release



Solution: (using the ***principle of impulse & momentum***)





Free body diagrams for ***motion*** up the plane

Newton's Law:

$$A: \quad +\nwarrow \sum F_y = N - W_A\cos(\theta) = 0 \quad \Rightarrow \quad f = \mu_k N = \mu_k W_A \cos(\theta)$$

Impulse and Momentum:

A: $L_{1x} + \sum (I_{1\to2})_x = L_{2x}$ with $L_{1x} = \left(\frac{W_A}{g}\right) v_{1x} = 0$ (released from rest)

$L_{2x} = \left(\frac{W_A}{g}\right) v_2$ and $\sum (I_{1\to2})_x = \big(T - W_A\sin(\theta) - \mu_k W_A\cos(\theta)\big)\Delta t$ (constant forces)

$$\Rightarrow \quad \left[T - W_A\sin(\theta) - \mu_k W_A\cos(\theta)\right]\Delta t = \left(\frac{W_A}{g}\right) v_2 \quad (1)$$

B: $L_{1x} + \sum (I_{1\to2})_x = L_{2x}$ with $L_{1x} = \left(\frac{W_B}{g}\right) v_{1x} = 0$ (released from rest)

$L_{2x} = \left(\frac{W_B}{g}\right) v_2$ (same velocity as A) and $\sum (I_{1\to2})_x = (W_B - T)\Delta t$ (constant forces)

$$\Rightarrow \quad \left[W_B - T\right]\Delta t = \left(\frac{W_B}{g}\right) v_2 \quad (2)$$

Adding equations (1) and (2) gives:

$$\left[W_B - W_A\sin(\theta) - \mu_k W_A\cos(\theta)\right]\Delta t = \left(\frac{W_A + W_B}{g}\right) v_2$$

$$\Rightarrow \quad v_2 = \left(\frac{g\,\Delta t}{W_A + W_B}\right)\left[W_B - W_A\sin(\theta) - \mu_k W_A\cos(\theta)\right] = 3.64989 \approx 3.65 \text{ (ft/s)}$$