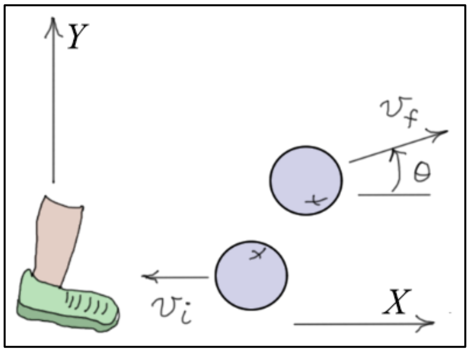
Elementary Dynamics Example #24: (Impulse & Momentum)

Given: $v_i = 5$ (ft/s), $v_f = 10$ (ft/s), $\theta = 30$ (deg)
$\Delta t = 0.01$ (sec), $W_B = 0.5$ (lb)

Find: a) average force exerted by the player on the ball
b) maximum force exerted by the player on the ball assuming a triangular force profile



Solution: (using the ***principle of impulse & momentum***)

a) $\boxed{\underset{\sim}{L}_1 + \underset{\sim}{I}_{1\to 2} = \underset{\sim}{L}_2}$ with $\boxed{\underset{\sim}{L}_1 = -\left(\frac{W_B}{g}\right) v_i \, \underset{\sim}{i}}$ and $\boxed{\underset{\sim}{L}_2 = \left(\frac{W_B}{g}\right) v_f \left(\cos(\theta)\,\underset{\sim}{i} + \sin(\theta)\,\underset{\sim}{j}\right)}$

X direction: $\boxed{-\left(\frac{W_B}{g}\right) v_i + \left(F_x\right)_{\text{avg}} \Delta t = \left(\frac{W_B}{g}\right) v_f \cos(\theta)}$

$\Rightarrow \boxed{\left(F_x\right)_{\text{avg}} = \left(\frac{W_B}{g\,\Delta t}\right)\left(v_i + v_f \cos(\theta)\right) \approx 21.2116 \approx 21.2 \text{ (lb)}}$

Y direction: $\boxed{0 + \left(F_y\right)_{\text{avg}} \Delta t = \left(\frac{W_B}{g}\right) v_f \sin(\theta)}$

$\Rightarrow \boxed{\left(F_y\right)_{\text{avg}} = \left(\frac{W_B}{g\,\Delta t}\right) v_f \sin(\theta) \approx 7.76398 \approx 7.76 \text{ (lb)}}$

$\boxed{|\underset{\sim}{F}_{\text{avg}}| \approx 22.6 \text{ (lb)}}$

Note: In the above calculation, we ***neglected*** the impulse of the ***weight force*** of the ball. If we include the weight force in our calculation, the equation in the Y direction becomes:

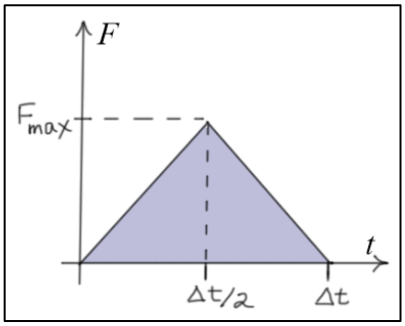
Y direction: $\boxed{\left(\left(F_y\right)_{\text{avg}} - W_B\right)\Delta t = \left(\frac{W_B}{g}\right) v_f \sin(\theta)}$

$\Rightarrow \boxed{\left(F_y\right)_{\text{avg}} = \left(\frac{W_B}{g\,\Delta t}\right) v_f \sin(\theta) + W_B \approx 8.26 \text{ (lb)}}$

$\boxed{|\underset{\sim}{F}_{\text{avg}}| \approx 22.8 \text{ (lb)}}$

b) If the force profile is assumed to be ***triangular***, the ***maximum*** force can be calculated. In this case, the impulse of the force is

$$\boxed{\int_0^{\Delta t} F dt = \tfrac{1}{2}\left(\Delta t\right) F_{\max}}$$



Ignoring the effect of the weight force gives

$\boxed{\left(F_x\right)_{\max} = \left(\frac{2W_B}{g\,\Delta t}\right)\left(v_i + v_f \cos(\theta)\right) \approx 42.4 \text{ (lb)}}$

$\boxed{\left(F_y\right)_{\max} = \left(\frac{2W_B}{g\,\Delta t}\right) v_f \sin(\theta) \approx 15.5 \text{ (lb)}}$

$\boxed{|\underset{\sim}{F}_{\max}| \approx 45.2 \text{ (lb)}}$

Using this approach, the ***maximum force*** is found to be ***twice*** the ***average force***.