

Elementary Dynamics Example #27: (Conservation of Momentum, Impact)

Given: $m_A = m_B = m$, $e = 0.6$,

m_A is released from rest at the top and strikes m_B at the bottom

Find: h the **maximum height** to which m_B swings

Solution: (**conservation of momentum and energy**)

- Using **conservation of energy**, the velocity of m_A as it strikes m_B is found.

$$\cancel{K}_1 + V_1 = K_2 + \cancel{V}_2 \quad (\text{datum at the bottom})$$

So,

$$V_1 = mgL = K_2 = \frac{1}{2}m(v_{A2})^2 \Rightarrow v_{A2} = \sqrt{2gL} \quad (\text{just before impact})$$

- Using conservation of momentum: (in X direction, 2 – just before impact, 3 – just after)

$$\cancel{m}(v_{A2}) + \underbrace{m(v_{B2})}_{\text{zero}} = \cancel{m}(v_{A3}) + \cancel{m}(v_{B3})$$

$$\Rightarrow v_{A3} + v_{B3} = v_{A2} = \sqrt{2gL}$$

Using the impact (restitution) equation:

$$\frac{v_{A3} - v_{B3}}{\cancel{v}_{B2} - v_{A2}} = e \Rightarrow v_{A3} - v_{B3} = -e v_{A2} = -0.6\sqrt{2gL}$$

Solving the two equations simultaneously gives:

$$v_{A3} = 0.2\sqrt{2gL}, \quad v_{B3} = 0.8\sqrt{2gL}$$

- Using conservation of energy for m_B after impact:

$$K_3 + \cancel{V}_3 = \cancel{K}_4 + V_4 \quad (\text{datum at the bottom})$$

So,

$$K_3 = \frac{1}{2}m(v_{B3})^2 = \frac{1}{2}m(0.8)^2(\cancel{2}gL) = 0.64mgL = V_4 = mgh \Rightarrow h = 0.64L$$

Question: What would the result be if $e = 1$?

Answer: Using the above approach, it is easy to show that, in this case, $h = L$. This makes sense if you recall that **kinetic energy** of the impact is **conserved** if $e = 1$.

