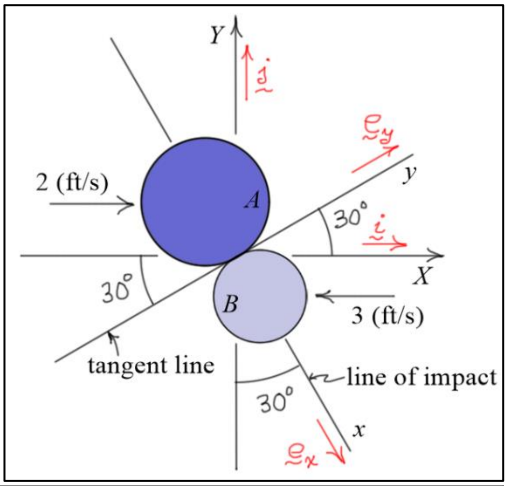
Elementary Dynamics Example #28: (Conservation of Momentum, Impact)



Given: $W_A = 13.2\times10^{-3}$ (lb), $W_B = 6.6\times10^{-3}$ (lb), $e = 0.65$

- velocities shown are just prior to collision
- disks are sliding on a smooth horizontal surface

Find: velocities of A and B just after impact

Solution:

- velocities are conserved in the $\underset{\sim}{e}_y$ direction
- linear momentum is conserved in the $\underset{\sim}{e}_x$ direction

Before impact: (state 1)

$$\boxed{\underset{\sim}{v}_A = 2\,\underset{\sim}{i} = 2\left(\sin(30)\underset{\sim}{e}_x + \cos(30)\underset{\sim}{e}_y\right) \text{ (ft/s)}}, \quad \boxed{\underset{\sim}{v}_B = -3\,\underset{\sim}{i} = -3\left(\sin(30)\underset{\sim}{e}_x + \cos(30)\underset{\sim}{e}_y\right) \text{ (ft/s)}}$$

Collision:

$\underset{\sim}{e}_y$ direction:

$$\boxed{\left(v_{Ay}\right)_2 = \left(v_{Ay}\right)_1 = 2\cos(30) \approx 1.7321} \quad \text{and} \quad \boxed{\left(v_{By}\right)_2 = \left(v_{By}\right)_1 = -3\cos(30) \approx -2.5981} \text{ (ft/s)}$$

$\underset{\sim}{e}_x$ direction: (conservation of linear momentum and restitution equation)

$$m_A\left(v_{Ax}\right)_2 + m_B\left(v_{Bx}\right)_2 = m_A\left(v_{Ax}\right)_1 + m_B\left(v_{Bx}\right)_1 = m_A\left(2\sin(30)\right) + m_B\left(-3\sin(30)\right)$$

$$\Rightarrow \boxed{m_A\left(v_{Ax}\right)_2 + m_B\left(v_{Bx}\right)_2 = \left(2m_A - 3m_B\right)\sin(30)}$$

Multiplying both sides by g gives:

$$\boxed{W_A\left(v_{Ax}\right)_2 + W_B\left(v_{Bx}\right)_2 = \left(2W_A - 3W_B\right)\sin(30) = 1.0248\times10^{-4}\,g}$$

$$\boxed{\frac{\left(v_{Bx}\right)_2 - \left(v_{Ax}\right)_2}{\left(v_{Ax}\right)_1 - \left(v_{Bx}\right)_1} = e} \quad \Rightarrow \quad \boxed{\begin{aligned} -\left(v_{Ax}\right)_2 + \left(v_{Bx}\right)_2 &= e\left[\left(v_{Ax}\right)_1 - \left(v_{Bx}\right)_1\right] \\ &= 0.65\left[2\sin(30) + 3\sin(30)\right] \approx 1.625 \end{aligned}}$$

Simultaneous equations:

$$\boxed{\begin{aligned} 13.2\left(v_{Ax}\right)_2 + 6.6\left(v_{Bx}\right)_2 &= 0.10248g \\ -\left(v_{Ax}\right)_2 + \left(v_{Bx}\right)_2 &= 1.625 \end{aligned}} \quad \Rightarrow \begin{cases} \left(v_{Ax}\right)_2 = -0.375 \text{ (ft/s)} \\ \left(v_{Bx}\right)_2 = \ \ 1.25 \text{ (ft/s)} \end{cases}$$

Notes:

- Incoming angle of each of the masses with the $\underset{\sim}{e}_y$ direction is equal to 30 degrees.
- Outgoing angles with the $\underset{\sim}{e}_y$ direction:

$$A: \boxed{\theta_A = \tan^{-1}\left(\frac{0.375}{1.7321}\right) \approx 12.2 \text{ (deg) CCW}} \quad B: \boxed{\theta_B = \tan^{-1}\left(\frac{1.25}{2.5981}\right) \approx 25.7 \text{ (deg) CCW}}$$

The smaller mass exits the collision at close to 30 degrees off the $\underset{\sim}{e}_y$ direction, but the larger mass is well below 30 degrees.