

Elementary Dynamics Example #29: (Rigid Body Kinematics – Pure Rotational Motion)

Given: $r_1 = 6$ (in), $r_2 = 4$ (in), $r_3 = 2$ (in)

$\omega_1 = 1000$ (rpm) CCW

$\alpha_1 = 5$ (r/s²) CCW

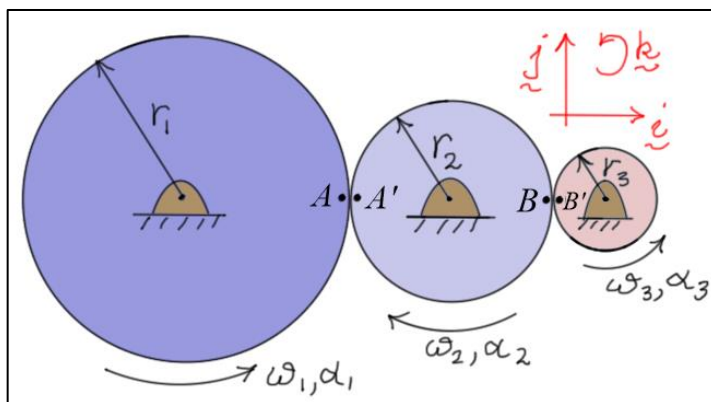
- no slipping between the gears

- radii represent the pitch circles

Find: a) ω_2, ω_3

b) α_2, α_3

c) $\underline{a}_A, \underline{a}_{A'}$



Solution:

a) Because the gears do not slip relative to each other, $\underline{v}_A = \underline{v}_{A'}$ and $\underline{v}_B = \underline{v}_{B'}$.

$$\underline{v}_A = r_1 \omega_1 \underline{j} = \underline{v}_{A'} = r_2 \omega_2 \underline{j} \Rightarrow \omega_2 = \left(\frac{r_1}{r_2}\right) \omega_1 = \frac{3}{2}(1000) = 1500 \text{ (rpm) CW}$$

$$\underline{v}_B = -r_2 \omega_2 \underline{j} = \underline{v}_{B'} = -r_3 \omega_3 \underline{j} \Rightarrow \omega_3 = \left(\frac{r_2}{r_3}\right) \omega_2 = \frac{4}{2}(1500) = 3000 \text{ (rpm) CCW}$$

Each of these rates can be expressed in radians/seconds by multiplying by $\frac{2\pi}{60}$. For example,

$$1000 \frac{\cancel{\text{rev}}}{\cancel{\text{min}}} \times \frac{1 \cancel{\text{min}}}{60 \text{ sec}} \times \frac{2\pi \text{ rad}}{1 \cancel{\text{rev}}} = 1000 \left(\frac{2\pi}{60}\right) \approx 104.7 \text{ (rad/s)}$$

b) Because the gears do not slip, the accelerations of the contact points *tangent* to the contacting surfaces must be equal. That is, $\left(\underline{a}_A\right)_{\text{tan}} = \left(\underline{a}_{A'}\right)_{\text{tan}}$ and $\left(\underline{a}_B\right)_{\text{tan}} = \left(\underline{a}_{B'}\right)_{\text{tan}}$.

$$\left(\underline{a}_A\right)_{\text{tan}} = r_1 \alpha_1 \underline{j} = \left(\underline{a}_{A'}\right)_{\text{tan}} = r_2 \alpha_2 \underline{j} \Rightarrow \alpha_2 = \left(\frac{r_1}{r_2}\right) \alpha_1 = \frac{6}{4}(5) = 7.5 \text{ (r/s}^2\text{) CW}$$

$$\left(\underline{a}_B\right)_{\text{tan}} = -r_2 \alpha_2 \underline{j} = \left(\underline{a}_{B'}\right)_{\text{tan}} = -r_3 \alpha_3 \underline{j} \Rightarrow \alpha_3 = \left(\frac{r_2}{r_3}\right) \alpha_2 = \frac{4}{2}(7.5) = 15 \text{ (r/s}^2\text{) CCW}$$

c) For A, $\underline{e}_r = \underline{i}$ and $\underline{e}_\theta = \underline{j}$. For A', $\underline{e}_r = -\underline{i}$ and $\underline{e}_\theta = \underline{j}$. Using the equations for radial and transverse acceleration for constant radius motion,

$$\underline{a}_A = -r_1 \omega_1^2 \underline{i} + r_1 \alpha_1 \underline{j} = \left(-\left(\frac{6}{12}\right)(104.72)^2\right) \underline{i} + \left(\left(\frac{6}{12}\right)5\right) \underline{j} \approx -5480 \underline{i} + 2.5 \underline{j} \text{ (ft/s}^2\text{)}$$

$$\underline{a}_{A'} = r_2 \omega_2^2 \underline{i} + r_2 \alpha_2 \underline{j} = \left(-\left(\frac{4}{12}\right)\left(\frac{1500(2\pi)}{60}\right)^2\right) \underline{i} + \left(\left(\frac{4}{12}\right)7.5\right) \underline{j} \approx 8225 \underline{i} + 2.5 \underline{j} \text{ (ft/s}^2\text{)}$$

Note that the accelerations of A and A' are *equal* only in the *tangential* direction, \underline{j} .

Their accelerations in the \underline{i} direction *differ* in both *magnitude* and *sign*.