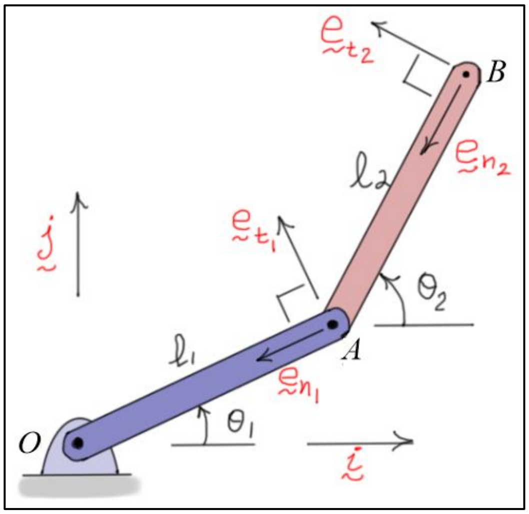
Elementary Dynamics Example #35: (Rigid Body Kinematics – Relative Acceleration)



Given: $\ell_1 = \ell_2 = 0.4$ (m), $\theta_1 = 25$ (deg), $\theta_2 = 60$ (deg)

$\omega_1 = \dot{\theta}_1 = 10$ (r/s) (CCW), $\omega_2 = \dot{\theta}_2 = 5$ (r/s) (CCW)

$\alpha_1 = \dot{\omega}_1 = 20$ (r/s^2) (CCW)

$\alpha_2 = \dot{\omega}_2 = -15$ (r/s^2) (CW)

Find: $\underset{\sim}{a}_B$

Solution:

Using the relative acceleration equation,

$$\boxed{\underset{\sim}{a}_B = \underset{\sim}{a}_A + \underset{\sim}{a}_{B/A}}$$

where

$$\begin{aligned}
\underset{\sim}{a}_A = \underset{\sim}{a}_{A/O} &= \left[\underset{\sim}{\alpha}_1 \times \underset{\sim}{r}_{A/O}\right] - \left[\omega_1^2 \underset{\sim}{r}_{A/O}\right] \\
&= \left[20\underset{\sim}{k} \times 0.4\left(\cos(25)\ \underset{\sim}{i} + \sin(25)\ \underset{\sim}{j}\right)\right] - 10^2(0.4)\left(\cos(25)\ \underset{\sim}{i} + \sin(25)\ \underset{\sim}{j}\right) \\
&= 8\left(-\sin(25)\ \underset{\sim}{i} + \cos(25)\ \underset{\sim}{j}\right) - 40\left(\cos(25)\ \underset{\sim}{i} + \sin(25)\ \underset{\sim}{j}\right) \\
&\approx -39.6333\ \underset{\sim}{i} - 9.65427\ \underset{\sim}{j}
\end{aligned}$$

$$\begin{aligned}
\underset{\sim}{a}_{B/A} &= \left[\underset{\sim}{\alpha}_2 \times \underset{\sim}{r}_{B/A}\right] - \left[\omega_2^2 \underset{\sim}{r}_{B/A}\right] \\
&= \left[-15\underset{\sim}{k} \times 0.4\left(\cos(60)\ \underset{\sim}{i} + \sin(60)\ \underset{\sim}{j}\right)\right] - 5^2(0.4)\left(\cos(60)\ \underset{\sim}{i} + \sin(60)\ \underset{\sim}{j}\right) \\
&= -6\left(-\sin(60)\ \underset{\sim}{i} + \cos(60)\ \underset{\sim}{j}\right) - 10\left(\cos(60)\ \underset{\sim}{i} + \sin(60)\ \underset{\sim}{j}\right) \\
&\approx 0.196152\ \underset{\sim}{i} - 11.6603\ \underset{\sim}{j}
\end{aligned}$$

$$\Rightarrow \boxed{\underset{\sim}{a}_B \approx -39.4\ \underset{\sim}{i} - 21.3\ \underset{\sim}{j}}\ \text{(m/s}^2\text{)}$$

Notes:

1. The terms $\underset{\sim}{\alpha}_1 \times \underset{\sim}{r}_{A/O}$ and $\underset{\sim}{\alpha}_2 \times \underset{\sim}{r}_{B/A}$ are along the $\underset{\sim}{e}_{t_1}$ and $\underset{\sim}{e}_{t_2}$ directions, respectively.
2. The terms $\left[-\omega_1^2 \underset{\sim}{r}_{A/O}\right]$ and $\left[-\omega_2^2 \underset{\sim}{r}_{B/A}\right]$ are along the $\underset{\sim}{e}_{n_1}$ and $\underset{\sim}{e}_{n_2}$ directions, respectively.