

Elementary Dynamics Example #36: (Rigid Body Kinematics – Relative Acceleration)

Given: ω, α, r, R

no slip between the fixed circular surface

(sun gear) and the planetary gear

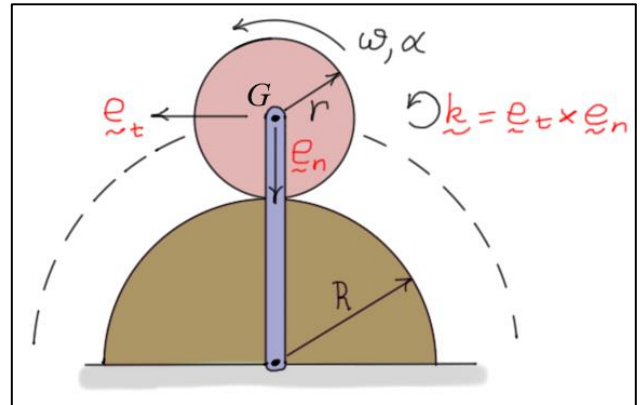
C is the contact point on the planetary gear

Find: $\underline{a}_G, \underline{a}_C$

Solution:

From a previous example on relative velocity,

$$\underline{v}_G = v \underline{e}_t = r\omega \underline{e}_t$$



Using normal and tangential components, the acceleration of G can be written as

$$\underline{a}_G = \dot{v} \underline{e}_t + \left(\frac{v^2}{\rho}\right) \underline{e}_n = r\dot{\omega} \underline{e}_t + \left(\frac{(r\omega)^2}{R+r}\right) \underline{e}_n = r\alpha \underline{e}_t + \left(\frac{(r\omega)^2}{R+r}\right) \underline{e}_n$$

Using the relative acceleration equation, the acceleration of C can be written as

$$\underline{a}_C = \underline{a}_G + \underline{a}_{C/G} \quad (C \text{ and } G \text{ are two points on the planetary gear})$$

where

$$\underline{a}_{C/G} = \left[\alpha \underline{k} \times \underline{r}_{C/G} \right] - \omega^2 \underline{r}_{C/G} = \left[\alpha \underline{k} \times r \underline{e}_n \right] - \omega^2 (r \underline{e}_n) = -r\alpha \underline{e}_t - r\omega^2 \underline{e}_n$$

Substituting the two results into the relative acceleration equation gives

$$\begin{aligned} \underline{a}_C &= (\cancel{r\alpha} - r\alpha) \underline{e}_t + \left(\frac{(r\omega)^2}{R+r} - r\omega^2\right) \underline{e}_n = \left(\frac{r^2\omega^2 - r\omega^2(R+r)}{R+r}\right) \underline{e}_n \\ &= \left(\frac{\cancel{r^2\omega^2} - \cancel{r^2\omega^2} - rR\omega^2}{R+r}\right) \underline{e}_n \quad \Rightarrow \quad \underline{a}_C = -\left(\frac{rR}{r+R}\right) \omega^2 \underline{e}_n \end{aligned}$$

As expected, the acceleration of C the contact point on the moving body (the planetary gear) is normal (perpendicular) to the contacting surfaces.