

Elementary Dynamics Example #38a: (Rigid Body Kinematics – Sliding Contact Acceleration)

Given: $v_A = 2 \text{ (ft/s)} = \text{constant}$, $\theta = 30 \text{ (deg)}$

Find: a) ω_{AB} , $\dot{\ell}$, b) α_{AB} , $\ddot{\ell}$

Solution #1: using the unit vectors $(\underline{i}, \underline{j}, \underline{k})$

a) Using the velocity equation for sliding contacts, write

$$\underline{v}_C = \underline{v}_A + (\underline{\omega}_{AB} \times \underline{r}_{C/A}) + \underline{v}_{C_{\text{rel}}} \quad * \quad (C \text{ is fixed, but moves on } AB)$$

Here,

$$\underline{v}_C = \underline{0} \quad \underline{v}_A = 2(-\cos(20)\underline{i} + \sin(20)\underline{j})$$

$$\underline{\omega}_{AB} \times \underline{r}_{C/A} = \omega_{AB} \underline{k} \times (4.5\underline{i} + 2.5\underline{j}) = \omega_{AB}(-2.5\underline{i} + 4.5\underline{j})$$

$$\underline{v}_{C_{\text{rel}}} = \dot{\ell} \underline{e}_1 = \dot{\ell}(4.5\underline{i} + 2.5\underline{j}) / \ell = \dot{\ell}(4.5\underline{i} + 2.5\underline{j}) / \sqrt{4.5^2 + 2.5^2} \quad (\text{velocity of } C \text{ on } AB)$$

Substituting into the velocity equation (*) gives the following scalar equations.

$$\begin{cases} 0 = -2\cos(20) - 2.5\omega_{AB} + \left(\frac{4.5}{\ell}\right)\dot{\ell} \\ 0 = 2\sin(20) + 4.5\omega_{AB} + \left(\frac{2.5}{\ell}\right)\dot{\ell} \end{cases} \quad \dots \text{solving gives } \Rightarrow \begin{cases} \omega_{AB} \approx -0.293458 \approx -0.293 \text{ (r/s)} \\ \dot{\ell} \approx 1.31068 \approx 1.31 \text{ (ft/s)} \end{cases}$$

b) Using the acceleration equation for sliding contacts, write

$$\underline{a}_C = \underline{a}_A + (\underline{\alpha}_{AB} \times \underline{r}_{C/A}) - \omega_{AB}^2 \underline{r}_{C/A} + \underline{a}_{C_{\text{rel}}} + 2(\underline{\omega}_{AB} \times \underline{v}_{C_{\text{rel}}}) \quad ** \quad (C \text{ is fixed, but moves on } AB)$$

Here,

$$\underline{a}_C = \underline{a}_A = \underline{0}$$

$$\underline{\alpha}_{AB} \times \underline{r}_{C/A} = \alpha_{AB} \underline{k} \times (4.5\underline{i} + 2.5\underline{j}) = \alpha_{AB}(-2.5\underline{i} + 4.5\underline{j}) \quad -\omega_{AB}^2 \underline{r}_{C/A} = -\omega_{AB}^2(4.5\underline{i} + 2.5\underline{j})$$

$$\underline{a}_{C_{\text{rel}}} = \ddot{\ell} \underline{e}_1 = \ddot{\ell}(4.5\underline{i} + 2.5\underline{j}) / \ell = \ddot{\ell}(4.5\underline{i} + 2.5\underline{j}) / \sqrt{4.5^2 + 2.5^2} \quad (\text{acceleration of } C \text{ on } AB)$$

$$2(\underline{\omega}_{AB} \times \underline{v}_{C_{\text{rel}}}) = 2\omega_{AB} \underline{k} \times \dot{\ell}(4.5\underline{i} + 2.5\underline{j}) / \ell = \left(\frac{2\omega_{AB}\dot{\ell}}{\ell}\right)(-2.5\underline{i} + 4.5\underline{j})$$

Substituting into the acceleration equation (***) gives the following scalar equations.

$$\begin{cases} 0 = -2.5\alpha_{AB} - 4.5\omega_{AB}^2 + \left(\frac{4.5}{\ell}\right)\ddot{\ell} - \left(\frac{5\omega_{AB}\dot{\ell}}{\ell}\right) \\ 0 = 4.5\alpha_{AB} - 2.5\omega_{AB}^2 + \left(\frac{2.5}{\ell}\right)\ddot{\ell} + \left(\frac{9\omega_{AB}\dot{\ell}}{\ell}\right) \end{cases} \quad \dots \text{solving gives } \Rightarrow \begin{cases} \alpha_{AB} \approx 0.149434 \approx 0.149 \text{ (r/s}^2\text{)} \\ \ddot{\ell} \approx 0.443318 \approx 0.443 \text{ (ft/s}^2\text{)} \end{cases}$$

Conclusions:

1. Bar AB is rotating clockwise ($\omega_{AB} < 0$), but slowing down ($\alpha_{AB} > 0$).
2. Point C is moving away from point A ($\dot{\ell} > 0$) at an increasing rate ($\ddot{\ell} > 0$).

