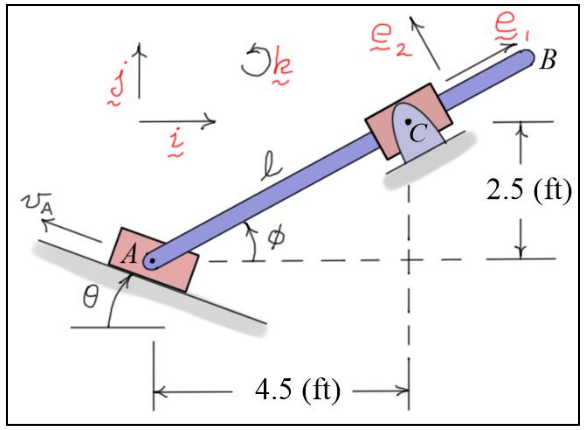
Elementary Dynamics Example #38b: (Rigid Body Kinematics – Sliding Contact Acceleration)



Given: $v_A = 2$ (ft/s) = constant , $\theta = 30$ (deg)

Find: a) ω_{AB} , $\dot{\ell}$, b) α_{AB} , $\ddot{\ell}$

Solution #2: using the unit vectors $\left(\underset{\sim}{e}_1, \underset{\sim}{e}_2, \underset{\sim}{k} \right)$

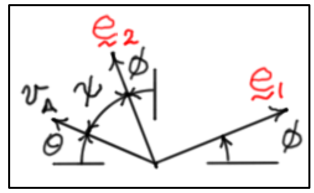
a) Using the velocity equation for sliding contacts, write (C is fixed, but moves on AB)

$$\boxed{\underset{\sim}{v}_C = \underset{\sim}{v}_A + \left(\underset{\sim}{\omega}_{AB} \times \underset{\sim}{r}_{C/A} \right) + \underset{\sim}{v}_{C_{\text{rel}}}} \quad ***$$

Here,

$$\boxed{\underset{\sim}{v}_C = \underset{\sim}{0}} \qquad \boxed{\underset{\sim}{\omega}_{AB} \times \underset{\sim}{r}_{C/A} = \omega_{AB} \underset{\sim}{k} \times \ell \underset{\sim}{e}_1 = \ell \omega_{AB} \underset{\sim}{e}_2}$$

$$\boxed{\underset{\sim}{v}_{C_{\text{rel}}} = \dot{\ell} \underset{\sim}{e}_1} \quad \text{(velocity of } C \text{ on } AB\text{)}$$



$$\boxed{\underset{\sim}{v}_A = 2\left(-\sin(\psi) \underset{\sim}{e}_1 + \cos(\psi) \underset{\sim}{e}_2 \right)} \qquad \boxed{\psi = 90 - \phi - \theta \approx 40.9454 \text{ (deg)}}$$

Substituting into the velocity equation (***) gives the following scalar equations.

$$\boxed{\begin{aligned} 0 &= -2\sin(\psi) + \dot{\ell} \\ 0 &= \ \ 2\cos(\psi) + \ell \omega_{AB} \end{aligned}} \quad \ldots\text{solving gives} \Rightarrow \boxed{\begin{cases} \omega_{AB} \approx -0.293458 \approx -0.293 \text{ (r/s)} \\ \quad \dot{\ell} \approx 1.31068 \approx 1.31 \text{ (ft/s)} \end{cases}}$$

b) Using the acceleration equation for sliding contacts, write

$$\boxed{\underset{\sim}{a}_C = \underset{\sim}{a}_A + \left(\underset{\sim}{\alpha}_{AB} \times \underset{\sim}{r}_{C/A} \right) - \omega_{AB}^2 \underset{\sim}{r}_{C/A} + \underset{\sim}{a}_{C_{\text{rel}}} + 2\left(\underset{\sim}{\omega}_{AB} \times \underset{\sim}{v}_{C_{\text{rel}}} \right)} \quad **** \text{ (} C \text{ is fixed, but moves on } AB\text{)}$$

where

$$\boxed{\underset{\sim}{a}_C = \underset{\sim}{a}_A = \underset{\sim}{0}} \qquad \boxed{\underset{\sim}{\alpha}_{AB} \times \underset{\sim}{r}_{C/A} = \alpha_{AB} \underset{\sim}{k} \times \ell \underset{\sim}{e}_1 = \ell \alpha_{AB} \underset{\sim}{e}_2} \qquad \boxed{-\omega_{AB}^2 \underset{\sim}{r}_{C/A} = -\ell \omega_{AB}^2 \underset{\sim}{e}_1}$$

$$\boxed{\underset{\sim}{a}_{C_{\text{rel}}} = \ddot{\ell} \underset{\sim}{e}_1} \quad \text{(acceleration of } C \text{ on } AB\text{)} \qquad \boxed{2\left(\underset{\sim}{\omega}_{AB} \times \underset{\sim}{v}_{C_{\text{rel}}} \right) = 2\omega_{AB} \underset{\sim}{k} \times \dot{\ell} \underset{\sim}{e}_1 = 2\omega_{AB} \dot{\ell} \underset{\sim}{e}_2}$$

Substituting into the acceleration equation (****) gives the following scalar equations.

$$\boxed{\begin{aligned} 0 &= -\ell \omega_{AB}^2 + \ddot{\ell} \\ 0 &= \ \ \ell \alpha_{AB} + 2\dot{\ell} \omega_{AB} \end{aligned}} \quad \ldots\text{solving gives} \Rightarrow \boxed{\begin{cases} \alpha_{AB} \approx 0.149434 \approx 0.149 \text{ (r/s}^2\text{)} \\ \ \ddot{\ell} \approx 0.443318 \approx 0.443 \text{ (ft/s}^2\text{)} \end{cases}}$$

Conclusions:

1. As expected, the results are the ***same*** as in Example #38a. The results ***do not depend*** on the unit vectors used to express the equations.
2. The unit vectors chosen for this second solution produce a ***less complicated set*** of scalar equations than those in Example #38a. A ***careful selection*** of unit vectors (directions) can often make the solution process ***less tedious***.