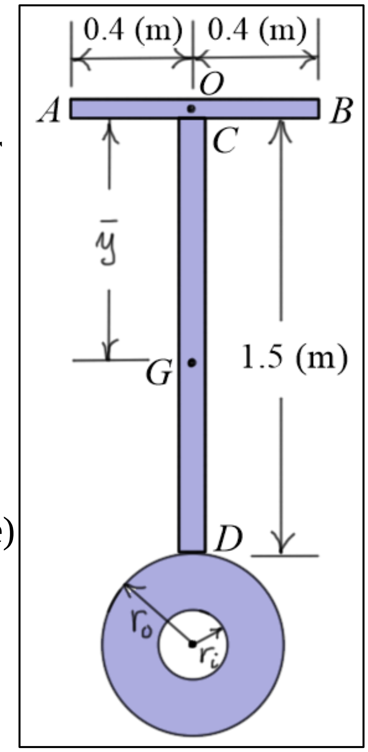
Elementary Dynamics Example #39: (Rigid Body Kinetics – Mass Center/Inertia)



Given: $m_{AB} = 2.4$ (kg), $m_{CD} = 4.5$ (kg), $\rho_{\text{plate}} = 12$ (kg/m^2)

$r_o = 0.3$ (m), $r_i = 0.1$ (m), G is the composite mass center

the circular plate is thin and the rods (AB and CD) are slender

Find: a) $\bar{y}$; b) I_G; c) I_O

Solution:

a) To find the location of G relative to the top bar AB, write

$$m_{AB}(0) + m_{CD}(0.75) + m_{\text{plate}}(1.8) = (m_{AB} + m_{CD} + m_{\text{plate}})\bar{y}$$

where

$$m_{\text{plate}} = \rho A = \rho\pi\left(r_o^2 - r_i^2\right) \approx 3.01593 \text{ (kg)}$$ (circular plate with hole)

So, the distance to the mass center is

$$\bar{y} = \left[\frac{m_{AB}(0) + m_{CD}(0.75) + m_{\text{plate}}(1.8)}{(m_{AB} + m_{CD} + m_{\text{plate}})}\right] \approx 0.887831 \text{ (m)}$$

b) The inertia of the composite shape about the composite mass center G is the sum of the inertias of the components about G.

$$I_G = \left(I_G\right)_{AB} + \left(I_G\right)_{CD} + \left(I_G\right)_{\text{plate}} \quad *$$

Using the parallel axes theorem for each of the components gives

$$\left(I_G\right)_{AB} = \tfrac{1}{12} m_{AB}(0.8)^2 + m_{AB}\,\bar{y}^2 \approx 2.01978 \text{ (kg-m}^2\text{)}$$

$$\left(I_G\right)_{CD} = \tfrac{1}{12} m_{CD}(1.5)^2 + m_{CD}\left(\bar{y} - 0.75\right)^2 \approx 0.929239 \text{ (kg-m}^2\text{)}$$

$$m_{\text{full plate}} = \rho A = \rho\pi r_o^2 \approx 3.39292 \text{ (kg)} \qquad m_{\text{hole}} = \rho A = \rho\pi r_i^2 \approx 0.376991 \text{ (kg)}$$

$$\left(I_G\right)_{\text{plate}} = \tfrac{1}{2} m_{\text{full plate}}\, r_o^2 - \tfrac{1}{2} m_{\text{hole}}\, r_i^2 + m_{\text{plate}}\left(1.8 - \bar{y}\right)^2 \approx 2.66021 \text{ (kg-m}^2\text{)}$$

Substituting into the inertia equation (*) gives

$$I_G \approx 5.60923 \approx 5.61 \text{ (kg-m}^2\text{)}$$

c) Using the parallel axes theorem on the composite shape, write

$$I_O = I_G + m_{\text{total}}\,\bar{y}^2 \approx 5.60923 + \left[(2.4 + 4.5 + 3.01593)(0.887831)^2\right] \approx 13.4254 \approx 13.4 \text{ (kg-m}^2\text{)}$$