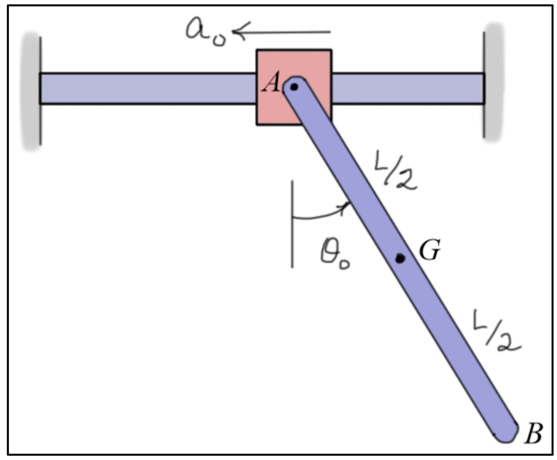
Elementary Dynamics Example #40: (Rigid Body Kinetics – Translation Example #1)



Given: $a_0 = 0.5\,g = \text{constant (ft/s}^2)$

$W_{\text{bar}} = W = 10 \text{ (lb.)}, \quad L = 2 \text{ (ft)}$

Find: constant angle, θ_0

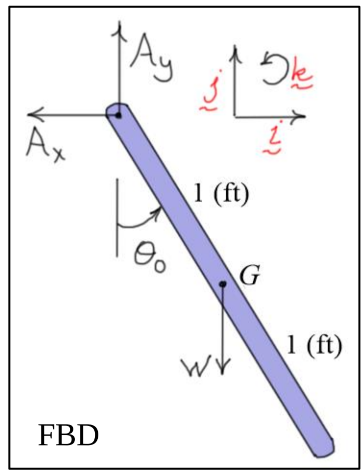
forces transmitted through the pin at A

Solution:

The bar translates under a constant acceleration of A, so the acceleration of G can be written as follows.

$$\boxed{\underset{\sim}{a}_G = \underset{\sim}{a}_A = -0.5\,g\,\underset{\sim}{i}}$$

Using the free-body-diagram and Newton's equations of motion, the following force and moment equations can be written.



$$\boxed{\overset{+}{\leftarrow}\sum F_x = A_x = \left(\frac{W}{\not g}\right)\left(0.5\not g\right) = 0.5W = 5 \text{ (lb)}}$$

$$+\!\uparrow \sum F_y = A_y - W = 0 \quad \Rightarrow \boxed{A_y = W = 10 \text{ (lb)}}$$

$$\circlearrowleft\!+ \sum M_G = \left(\left(\tfrac{L}{2}\right)\cos(\theta_0)\right)A_x - \left(\left(\tfrac{L}{2}\right)\sin(\theta_0)\right)A_y = 0$$

$$\Rightarrow 5\cos(\theta_0) - 10\sin(\theta_0) = 0 \quad \Rightarrow \tan(\theta_0) = \frac{\sin(\theta_0)}{\cos(\theta_0)} = \frac{5}{10}$$

$$\Rightarrow \boxed{\theta_0 \approx \begin{cases} 26.6 \text{ (deg)} \\ 26.6 + 180 = 206.6 \text{ (deg)} \end{cases}}$$

So, the bar is either ***lagging*** and ***below*** A $\left(26.6 \text{ (deg)}\right)$ or is ***leaning forward*** and ***above*** A $\left(206.6 \text{ (deg)}\right)$.

The moment equation could also be written about A.

$$\circlearrowleft\!+ \sum M_A = -\left(\left(\tfrac{L}{2}\right)\sin(\theta_0)\right)W = -\left(\left(\tfrac{L}{2}\right)\cos(\theta_0)\right)m\,a_G = -\left(\left(\tfrac{L}{2}\right)\cos(\theta_0)\right)\left(\frac{W}{\not g}\right)\left(0.5\not g\right)$$

$$\boxed{\frac{\sin(\theta_0)}{\cos(\theta_0)} = \tan(\theta_0) = 0.5} \quad \ldots \text{ same result}$$