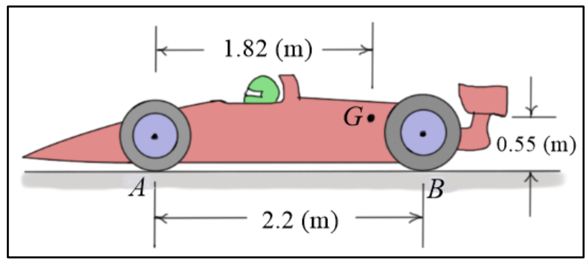
Elementary Dynamics Example #41: (Rigid Body Kinetics – Translation Example #2)

Given: physical dimensions, $m = 975\ \text{(kg)}$, $\mu_s = 0.8$
neglect mass/inertia of tires
assume four-wheel drive

Find: $a_{\max}$, the maximum acceleration so the front wheels do not leave the ground and the tires do not slip



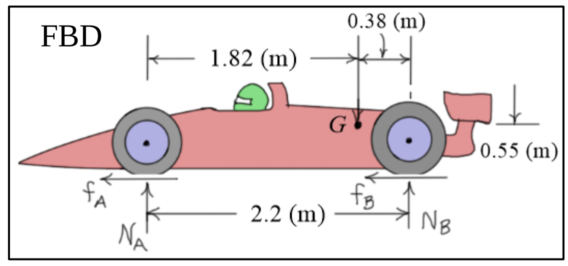
Solution:

One of the two conditions will ***limit*** the ***maximum acceleration***, but the specific condition is not known a priori. So, assumptions will be made and then checked.

Assumption 1: all four tires are just ready to slip

In this case, the friction and normal forces on the front and rear tires are related through the coefficient of static friction. That is,

$$\boxed{f_A = \mu_s N_A} \quad \boxed{f_B = \mu_s N_B}$$



Using Newton's laws of motion and the free-body diagram, write

$$\boxed{\overset{+}{\leftarrow} \sum F_x = 0.8 N_A + 0.8 N_B = m\, a_{\max}} \quad \boxed{+\!\uparrow \sum F_y = N_A + N_B - W = 0}$$

$$\boxed{\circlearrowleft\!+ \sum M_G = 0.38 N_B - 1.82 N_A - 0.55\left(f_A + f_B\right) = 0}$$

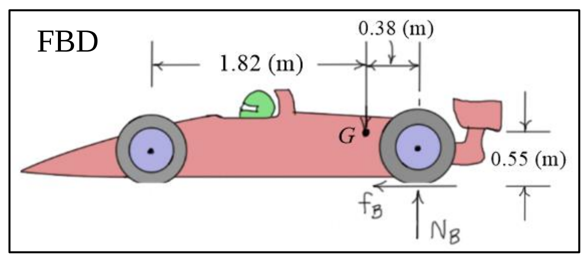
Substituting into the moment equation for the friction forces in terms of the normal forces, the last two equations can be solved for the normal forces.

$$\boxed{\begin{array}{l} \quad\;\; N_A \;+\; N_B = W = 975 \times 9.81 = 9564.75 \\ -2.26 N_A - 0.06 N_B = 0 \end{array}} \;\Rightarrow\; \boxed{\begin{cases} N_A = -261\ \text{(N)} \\ N_B = 9826\ \text{(N)} \end{cases}}$$

The normal forces cannot be negative, so the front wheels will lift off the ground before the tires begin to slip.

Assumption 2: front wheels just begin to lift off the ground

In this condition, the total normal force on the front wheels is zero. That is,



$$\boxed{N_A = f_A = 0}$$

Using Newton's laws of motion and the modified free-body-diagram, write

$$\boxed{\overset{+}{\leftarrow} \sum F_x = f_B = m\, a_{\max}} \quad \boxed{+\!\uparrow \sum F_y = N_B - W = 0} \;\Rightarrow\; \boxed{N_B = W = 9564.75\ \text{(N)}}$$

$$\boxed{\circlearrowleft\!+ \sum M_G = 0.38 N_B - 0.55 f_B = 0}$$

Having solved the second equation for N_B, substitute that value into the third equation to find f_B, and substitute f_B into the first equation to find $a_{\max}$.

$$\boxed{f_B = \left(\tfrac{0.38}{0.55}\right) N_B = 6608.4 \approx 6610\ \text{(N)}} \quad \boxed{a_{\max} = f_B / m \approx 6.78\ \text{(m/s}^2\text{)}}$$

In this case, the friction and normal forces at *B* are ***not related*** through the coefficient of friction, and as should be expected, $\boxed{f_B = 6610\ \text{(N)} < f_{\max} = \mu_s N_B = 7652\ \text{(N)}}$.

To get the above result, multiple equations were solved to find $a_{\max}$. It could have been found with a single equation by taking moments about *B*. Specifically,

$$\boxed{\circlearrowleft\!+ \sum M_B = 0.38 W = 0.55\left(m\, a_{\max}\right)} \;\Rightarrow\; \boxed{a_{\max} = \frac{0.38\, \cancel{m}\, g}{0.55\, \cancel{m}} \approx 6.78\ \text{(m/s}^2\text{)}}$$

Notes:

- It is often necessary to ***assume*** something about the motion of a body when ***friction*** is involved. After solving the problem based on that assumption, the results must be ***checked*** to ensure the ***validity*** of the assumption. If the assumption is valid, the results stand. If the assumption is not valid, then the problem must be ***solved again*** based on a different assumption. After solving, that assumption must also be checked.
- In the above example, for assumption #1 to be true, the ***normal forces*** on the front and rear wheels must be ***greater than zero***. The ground can ***support*** the wheels, but it cannot hold them down.
- For assumption #2 to be true, the friction force at *B* must be ***less than*** the ***maximum allowable friction force***. That is, $\boxed{f_B < f_{\max} = \mu_s N_B}$.