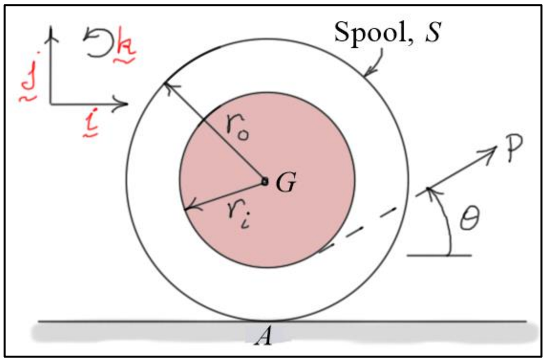
Elementary Dynamics Example #43: (Rigid Body Kinetics – General Plane Motion #1)



Given: $r_o = 0.4$ (m), $r_i = 0.25$ (m), $m = 100$ (kg)

$P = 200$ (N), $\theta = 20$ (deg), $k_G = 0.3$ (m)

$\mu_s = 0.2$, $\mu_k = 0.15$

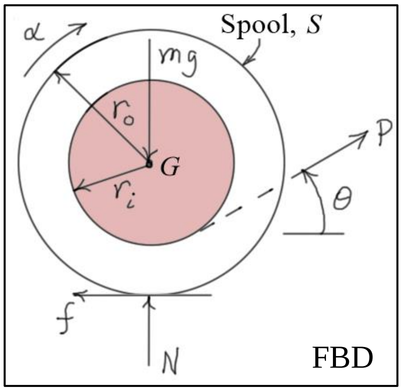
Find: $\underset{\sim}{\alpha}$, the angular acceleration of the spool S

Solution:

There are two possibilities – either the spool ***slips***, or it ***does not slip***.

Assumption: S ***rolls without slipping*** on the horizontal plane

In this case, the acceleration of G is directly related to the angular acceleration of S.



$\boxed{\underset{\sim}{a}_G = r_o \alpha \underset{\sim}{i}}$ (assuming that α is positive clockwise)

Using the free-body-diagram and Newton's laws of motion, write

$$\xrightarrow{+} \sum F_x = P\cos(20) - f = m a_G = m r_o \alpha$$

$$+\!\uparrow \sum F_y = N + P\sin(20) - mg = 0 \quad \Rightarrow \quad N = mg - P\sin(20) = 182.519 \approx 183 \text{ (N)}$$

$$\circlearrowright\!+ \sum M_G = r_o f - r_i P = I_G \alpha = m k_G^2 \alpha$$

The first and third equations can be solved simultaneously for f and α.

$$\begin{aligned} f + (100 \times 0.4)\alpha &= P\cos(20) = 200\cos(20) \\ 0.4 f - (100 \times 0.3^2)\alpha &= r_i P = 50 \end{aligned} \quad \Rightarrow \quad \begin{cases} f = 147.658 \approx 148 \text{ (N)} \\ \alpha = 1.00702 \approx 1.00 \text{ (r/s}^2\text{)} \end{cases}$$

Check: The signs of the friction force and the angular acceleration are consistent with the assumption. Also, $\boxed{f \approx 148 \text{ (N)} < f_{\max} \approx 183 \text{ (N)}}$, so the spool does not slip.

Notes:

- If the spool slips, $f = \mu_k N$ and $a_G \neq r_o \alpha$. In this case, the ***friction*** and ***normal forces*** are ***directly related***, but the ***acceleration*** of G and the ***angular acceleration*** of the spool are ***not***. If this assumption is made, you should find the conditions of the assumption are ***violated***. You may want to try this!
- Some find the above results ***troubling***. But, in fact, the spool does ***roll*** to the ***right***. If the angle θ is increased to 90 (deg), the spool will ***roll*** to the ***left***. It is not difficult to find the angle θ_t for which the spool will not roll. In this case, it simply ***translates*** to the right. $\left(\text{Ans: } \theta_t \approx 47.6 \text{ (deg)}\right)$