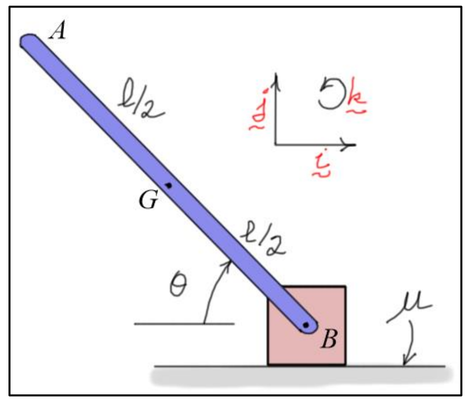
Elementary Dynamics Example #44: (Rigid Body Kinetics – General Plane Motion #2)



Given: $\ell = 2$ (ft), $W_{AB} = 4$ (lb), $W_B = 10$ (lb)

system is released from rest at $\theta = 45$ (deg)

Find: a_B, the acceleration of block B at the instant of release

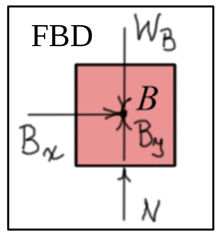
Solution:

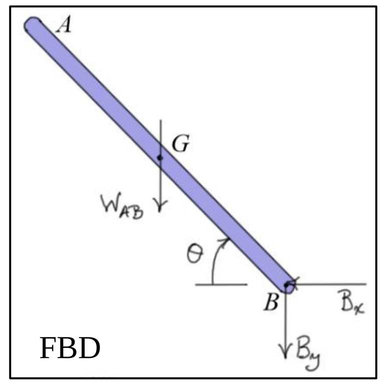
Writing Newton's laws of motion using the free-body diagrams gives

Block *B*:

$$\xrightarrow{+} \sum F_x = B_x = \left(\tfrac{W_B}{g}\right) a_B \qquad (1)$$

$$+\uparrow \sum F_y = B_y + N - W_B = 0 \qquad (2)$$





Bar *AB*:

$$\xrightarrow{+} \sum F_x = -B_x = \left(\tfrac{W_{AB}}{g}\right) a_{G_x} \qquad (3)$$

$$+\uparrow \sum F_y = -B_y - W_{AB} = \left(\tfrac{W_{AB}}{g}\right) a_{G_y} \qquad (4)$$

$$\circlearrowleft + \sum M_G = -\left(\tfrac{\ell}{2}\sin(45)\right)B_x - \left(\tfrac{\ell}{2}\cos(45)\right)B_y = I_G\alpha = \tfrac{1}{12}\left(\tfrac{W_{AB}}{g}\right)\ell^2\alpha \qquad (5)$$

These are five equations in seven unknowns ($B_x, B_y, N, a_{G_x}, a_{G_y}, \alpha, a_B$). Two more equations are required. A kinematic analysis provides these equations.

Kinematics:

The angular acceleration α, the acceleration a_B, and the components of acceleration $\underset{\sim}{a}_G$ can be related using the concept of relative acceleration.

$$\underset{\sim}{a}_G = \underset{\sim}{a}_B + \underset{\sim}{a}_{G/B} = a_B\,\underset{\sim}{i} + \left(\alpha\,\underset{\sim}{k} \times \underset{\sim}{r}_{G/B}\right) - \cancel{\omega_{AB}^2}\,\underset{\sim}{r}_{G/B} = \left(a_B\,\underset{\sim}{i}\right) + \alpha\,\underset{\sim}{k} \times \left(-\tfrac{\ell}{2}\cos(45)\,\underset{\sim}{i} + \tfrac{\ell}{2}\sin(45)\,\underset{\sim}{j}\right)$$

$$= \left(a_B - \tfrac{\ell}{2}\alpha\sin(45)\right)\underset{\sim}{i} + \left(-\tfrac{\ell}{2}\alpha\cos(45)\right)\underset{\sim}{j}$$

$$\Rightarrow \boxed{a_{G_x} = a_B - \tfrac{\ell}{2}\alpha\sin(45)} \qquad \boxed{a_{G_y} = -\tfrac{\ell}{2}\alpha\cos(45)} \qquad (6)$$

Using Eq. (6), a_{G_x} and a_{G_y} can be eliminated from Eqs. (3) and (4) to get four equations in four unknowns.

$$\begin{aligned} B_x - \left(\tfrac{W_B}{g}\right)a_B &= 0 \\ -B_x - \left(\tfrac{W_{AB}}{g}\right)\left(a_B - \tfrac{\ell}{2}\alpha\sin(45)\right) &= 0 \\ -B_y - \left(\tfrac{W_{AB}}{g}\right)\left(-\tfrac{\ell}{2}\alpha\cos(45)\right) &= W_{AB} \\ -\left(\tfrac{\ell}{2}\sin(45)\right)B_x - \left(\tfrac{\ell}{2}\cos(45)\right)B_y - \tfrac{1}{12}\left(\tfrac{W_{AB}}{g}\right)\ell^2\alpha &= 0 \end{aligned}$$

… solving gives

$$\begin{aligned} B_x &= 1.2 \text{ (lb)} \\ B_y &= -2.32 \text{ (lb)} \\ a_B &= 3.86 \text{ (ft/s}^2\text{)} \rightarrow \\ \alpha &= 19.1 \text{ (r/s}^2\text{)} \circlearrowleft \end{aligned}$$