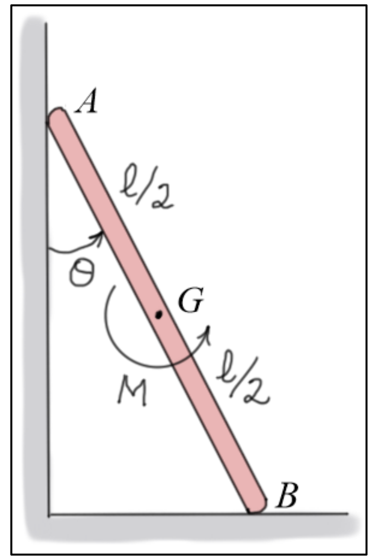
Elementary Dynamics Example #45a: (Rigid Body Kinetics – Work and Energy #1)



Given: $\ell = 4$ (m), $m = 18$ (kg), $M = 50$ (N-m) = constant

released from rest at $\theta = 10$ (deg), neglect friction

Find: ω the angular velocity of AB when $\theta = 40$ (deg)

Solution: position 1 is $\theta = 10$ (deg), and position 2 is $\theta = 40$ (deg)

$$\boxed{KE_1 + U_{1\to2} = KE_2}$$

Here,

$$\boxed{KE_1 = 0} \text{ (released from rest)}$$

$$\boxed{KE_2 = \tfrac{1}{2} I_{IC}\omega^2 = \tfrac{1}{2}\left(\tfrac{1}{3}m\ell^2\right)\omega^2 = \tfrac{1}{6}m\ell^2\omega^2 = 48\omega^2}$$

$$U_{1\to2} = \left(U_{1\to2}\right)_{\text{gravity}} + \left(U_{1\to2}\right)_M = V_1 - V_2 + M\left(\Delta\theta\right)$$

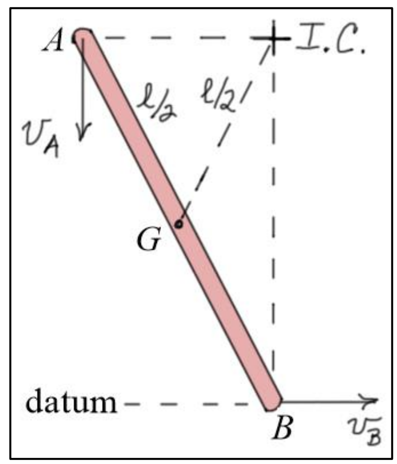
$$= mg\left(\tfrac{\ell}{2}\right)\cos(10) - mg\left(\tfrac{\ell}{2}\right)\cos(40) + 50(40-10)\tfrac{\pi}{180}$$

$$= 77.2585 + 26.1799$$

$$\Rightarrow \boxed{U_{1\to2} = 103.438 \text{ (N-m)}}$$

Solving,

$$48\omega^2 = 103.438 \quad \Rightarrow \boxed{\omega \approx 1.47 \text{ (r/s)}} \circlearrowleft$$



Notes:

1. The ***instantaneous center*** (IC) is used to calculate the kinetic energy of AB. Compare this to using the ***general formula*** $KE = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$. To use the general formula in this case, the mass-center velocity v_G must be written in terms of the angular velocity ω.
2. This problem becomes much more difficult if ***friction*** is included. Why?