

Elementary Dynamics Example #46: (Rigid Body Kinetics – Conservation of Energy)

Given: AB and BC are slender rods, $\ell = 3$ (ft)

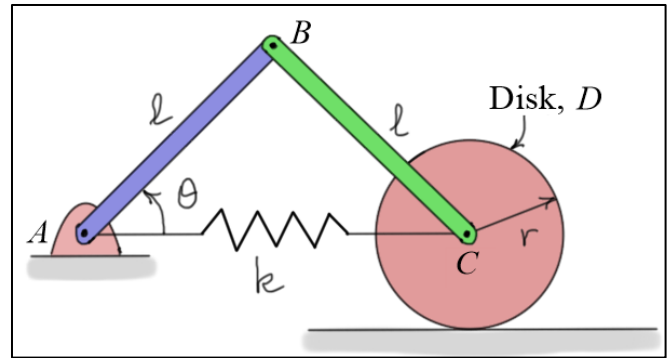
$$W_{AB} = W_{BC} = W = 15 \text{ (lb)}, W_D = 20 \text{ (lb)}$$

$$k = 4 \text{ (lb/ft)}$$

spring is unstretched at $\theta = 45$ (deg)

released from rest at $\theta = 45$ (deg)

Disk rolls without slipping, $r = 1$ (ft)



Find: ω_{AB} the angular velocity of AB at $\theta = 0$ (deg)

Solution:

Using the **conservation of energy** on the system (state 1: $\theta = 45$ (deg); state 2: $\theta = 0$ (deg))

$$\boxed{K_1 + V_1 = K_2 + V_2} \quad (\text{released from rest})$$

zero

Defining a datum at the horizontal line through A and C gives

$$\boxed{V_1 = \underbrace{(V_1)_{\text{gravity}}}_{\text{zero}} + \underbrace{(V_1)_{\text{spring}}}_{\text{zero}} = 2\left(W \frac{\ell}{2} \sin(45)\right) \approx 31.8198 \text{ (ft-lb)}}$$

$$\boxed{V_2 = \underbrace{(V_2)_{\text{gravity}}}_{\text{zero}} + (V_2)_{\text{spring}} = \frac{1}{2}k(2\ell - 2\ell \cos(45))^2 = 2k\ell^2(1 - \cos(45))^2 \approx 6.17662 \text{ (ft-lb)}}$$

In state 2, the disk has **zero velocity**, AB is rotating about A , and the **instantaneous center** of BC is at C . Also, due to the **symmetry** of the mechanism, the angular velocities of AB and BC are **equal** (but in opposite directions).

$$\boxed{K_2 = (K_2)_{\text{disk}} + (K_2)_{AB} + (K_2)_{BC} = \underbrace{0}_{(v_C)_2=0} + \frac{1}{2}I_A \omega_{AB}^2 + \frac{1}{2}I_C \omega_{BC}^2}$$

$$= \frac{1}{2}\left(\frac{1}{3}\left(\frac{W}{g}\right)\ell^2\right)\omega_{AB}^2 + \frac{1}{2}\left(\frac{1}{3}\left(\frac{W}{g}\right)\ell^2\right)\omega_{AB}^2 = \frac{1}{3}\left(\frac{W}{g}\right)\ell^2\omega_{AB}^2 \approx 1.39752\omega_{AB}^2$$

Substituting into the conservation of energy equation gives

$$31.8198 = 6.17662 + 1.39752\omega_{AB}^2 \Rightarrow \omega_{AB} = \sqrt{\frac{31.8198 - 6.17662}{1.39752}} \approx 4.28 \text{ (rad/s)}$$

The motion of AB is **clockwise**, and the motion of BC is **counter-clockwise**.

Question: What other forces (besides the spring and gravity) affect the motion of this system?

Why is the work done by these forces on the system equal to zero?