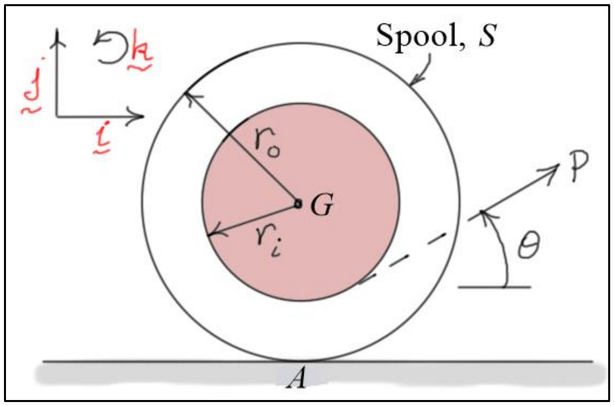
Elementary Dynamics Example #47: (Rigid Body Kinetics – Impulse & Momentum #1)



Given: $r_o = 0.4$ (m), $r_i = 0.25$ (m), $m = 100$ (kg)

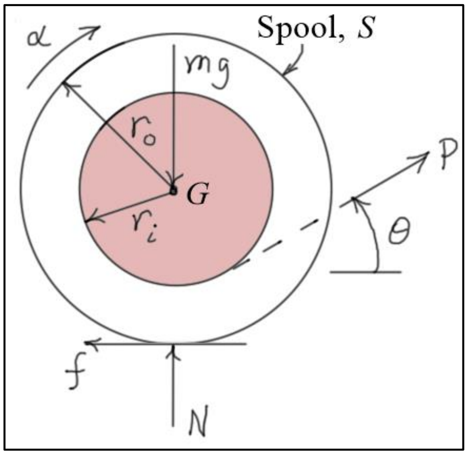
$k_G = 0.3$ (m), $P = 200$ (N) , $\theta = 20$ (deg)

released from ***rest*** with $\mu_s = 0.2$, $\mu_k = 0.15$

Find: ω, the angular velocity of S after 3 seconds

Solution:

The spool is ***released from rest*** and when the force P is applied all ***reaction forces*** are ***constant***. Applying the principles of linear and angular impulse and momentum to the free body diagram gives



$$\underbrace{\underset{\sim}{L}_1}_{\text{zero}} + \sum \int \underset{\sim}{F} dt = \underset{\sim}{L}_2 \qquad \underbrace{\left(\underset{\sim}{H}_G\right)_1}_{\text{zero}} + \sum \int \underset{\sim}{M}_G \, dt = \left(\underset{\sim}{H}_G\right)_2$$

$$\overset{+}{\rightarrow} \sum \int F_x dt = \left(P\cos(\theta) - f\right)\Delta t = m\left(v_G\right)_{2x} \qquad (1)$$

$$+\!\uparrow \sum \int F_y dt = \left(P\sin(\theta) + N - mg\right)\Delta t = m\left(v_G\right)_{2y} = 0 \qquad (2)$$

$$\circlearrowright\!+ \sum \int M_G dt = \left(r_o\, f - r_i\, P\right)\Delta t = I_G \omega_2 = m k_G^2\, \omega_2 \qquad (3)$$

Assuming the spool ***rolls without slipping***, $\left(v_G\right)_{2x}$ and ω_2 are related as follows.

$$\left(v_G\right)_{2x} = r_o\, \omega_2$$

After ***substituting*** for $\left(v_G\right)_{2x}$ in Eq. (1) and ***rearranging*** terms, Eqs. (1) and (3) can be written

$$\begin{aligned} \left(m\, r_o\right)\omega_2 \quad + \left(\Delta t\right) f &= P\cos(\theta)\Delta t \approx 563.8156 \\ \left(m\, k_G^2\right)\omega_2 - \left(r_o\, \Delta t\right) f &= -r_i\, P\, \Delta t = -150 \end{aligned}$$

Solving gives: $\omega_2 \approx 3.02$ (rad/s) (clockwise) $f \approx 147.658 \approx 148$ (lb)

Check: Using Eq. (2), $f_{\max} = \mu_s N = \mu_s\left(mg - P\sin(\theta)\right) \approx 183 \text{ (N)} > f$ $\Rightarrow$ no slipping occurs

Note:

To ***apply*** the principles of ***linear*** and ***angular impulse and momentum*** to a problem, the ***impulses*** of the forces and moments must be calculated. If the forces and moments are ***position*** (and hence, ***time***) ***dependent***, it may be ***quite difficult*** to calculate the linear and angular impulses. In these cases, it may ***not*** be ***practical*** to use these principles.