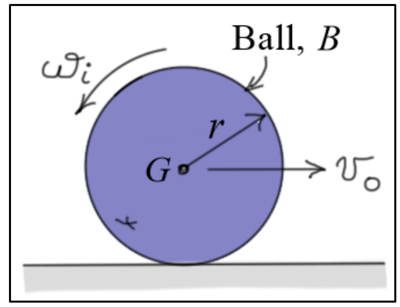
Elementary Dynamics Example #49: (Rigid Body Kinetics – Impulse & Momentum #3)



Given: $m_{\text{ball}} = m = 5$ (kg); $r = 0.1$ (m); $\mu_k = 0.08$

initial backspin, $\omega_i = 10$ (rad/s) (counterclockwise)

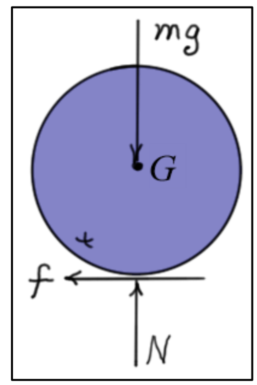
initial velocity, $v_o = 5$ (m/s) (to the right)

Find: a) Δt_b, the time it takes for ball to stop back spinning and the corresponding velocity of G

b) Δt_r, the time it takes for ball to start rolling without slipping and the corresponding velocity of G

Solution:

The summation of vertical forces gives $N = mg$. Because the ball is slipping during the entire motion, $f = \mu_k N = \mu_k mg$.



a) Applying the principles of linear and angular momentum gives

$$\xrightarrow{+} (L_1)_x + \sum (\text{Imp})_x = (L_2)_x$$ (1: initial state, 2: backspin stops)

$$(mv_G)_1 - f\,\Delta t_b = (mv_G)_2$$

$$\circlearrowleft^{+}\ (\underset{\sim}{H}_G)_1 + \sum \int \underset{\sim}{M}_G\, dt = (\underset{\sim}{H}_G)_2 \qquad I_G = \tfrac{2}{5} m r^2 = \tfrac{2}{5} \times 5 \times 0.1^2 = 0.02 \text{ (kg-m}^2\text{)}$$

$$I_G\,\omega_1 + r\, f\, \Delta t_b = I_G\,\omega_2 = 0$$ (angular velocity is ***zero*** at instant back spinning stops)

$$0.02(-10) + 0.1\mu_k\, m\, g\, \Delta t_b = 0 \quad \Rightarrow \quad \Delta t_b = \frac{0.2}{(0.1)(0.08)(5)(9.81)} \approx 0.50968 \approx 0.510 \text{ (sec)}$$

Substituting for Δt_b in the equation for linear momentum gives

$$(v_G)_2 = \tfrac{1}{m}(mv_o - f\,\Delta t_b) = v_o - \mu_k g \Delta t_b = 5 - (0.08 \times 9.81 \times \Delta t_b) = 4.6 \text{ (m/s)}$$

b) (1: initial state, 3: ball begins to roll without slipping) $\Rightarrow (v_G)_3 = r\,\omega_3$

$$(mv_G)_1 - f\,\Delta t_r = (mv_G)_3 \quad \Rightarrow \quad 5(v_G)_3 + (0.08 \times 5 \times 9.81)\Delta t_r = mv_o = 25$$

$$I_G\,\omega_1 + r\, f\, \Delta t_r = I_G\,\omega_3 \quad \Rightarrow \quad \left(\tfrac{0.02}{0.1}\right)(v_G)_3 - (0.1 \times 0.08 \times 5 \times 9.81)\Delta t_r = 0.02 \times (-10) = -0.2$$

Solving the last two equations simultaneously gives

$$(v_G)_3 \approx 3.28571 \approx 3.29 \text{ (m/s)}$$
$$\Delta t_r \approx 2.18436 \approx 2.18 \text{ (sec)}$$