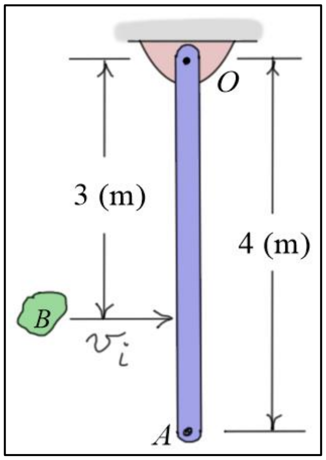
Elementary Dynamics Example #50: (Rigid Body Kinetics – Impulse & Momentum – Impact #1)

Given: bar OA is initially at rest when the blob B strikes it

B sticks to the bar and they move together after impact

$m_{OA} = 15$ (kg); $m_B = 5$ (kg); $v_i = 10$ (m/s)

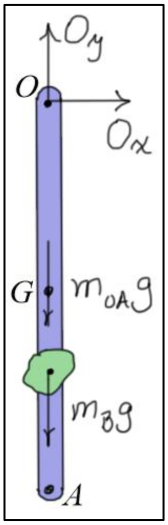
Find: θ_{max}, the maximum angle the bar reaches after impact



Solution:

During the impact, $\sum M_O = 0$, so the ***angular momentum*** of the ***system*** about O is ***conserved***.

$$\circlearrowleft \left(\underset{\sim}{H}_O\right)_1 = \left(\underset{\sim}{H}_O\right)_2$$ (state 1: just before impact; state 2: just after impact)



Here,

$$\left(\underset{\sim}{H}_O\right)_1 = 3\left(m_B v_i\right) = 3 \times 5 \times 10 = 150 \text{ (N-m-s)}$$

$$\left(\underset{\sim}{H}_O\right)_2 = 3\left(m_B v_B\right)_2 + I_O\omega_2 = 3m_B\left(3\omega_2\right) + \left(\tfrac{1}{3}m_{OA}\ell^2\right)\omega_2 = \left(45 + \left(\tfrac{1}{3}\times 15 \times 4^2\right)\right)\omega_2 = 125\omega_2$$

Substituting into the conservation of angular momentum equation gives

$$\omega_2 = 150/125 = 1.2 \text{ (rad/s)}$$ (angular velocity of OA just after impact)

To find the ***maximum angle*** the bar reaches, apply the ***conservation of energy***. Defining the ***datum*** of the ***bar*** at the bottom-most position of G, and the ***datum*** of ***B*** at its bottom-most position and observing the ***kinetic energy*** of the system is ***zero*** at the ***maximum angle***, write

$$KE_2 + \underbrace{V_2}_{\text{zero}} = \underbrace{KE_3}_{\text{zero}} + V_3$$ (state 2: just after impact; state 3: at maximum angle)

Here,

$$KE_2 = \tfrac{1}{2}I_O\omega_2^2 + \tfrac{1}{2}m_B\left(v_B^2\right)_2 = \tfrac{1}{2}\left(\tfrac{1}{3}m_{OA}\ell^2\right)\omega_2^2 + \tfrac{1}{2}m_B\left(3\omega_2\right)^2 = \tfrac{1}{2}(80)(1.2)^2 + \tfrac{9}{2}(5)(1.2)^2 = 90 \text{ (N-m)}$$

$$V_3 = m_{OA}g\left(2 - 2\cos(\theta_{max})\right) + m_B g\left(3 - 3\cos(\theta_{max})\right) = \left(2m_{OA} + 3m_B\right)g\left(1 - \cos(\theta_{max})\right) = 45g\left(1 - \cos(\theta_{max})\right)$$

Substituting and solving:

$$45g\left(1 - \cos(\theta_{max})\right) = 90 \quad \Rightarrow 1 - \cos(\theta_{max}) = 90/\left(45g\right) \quad \Rightarrow \theta_{max} = \cos^{-1}\left(\frac{45g - 90}{45g}\right)$$

$$\Rightarrow \boxed{\theta_{max} \approx 37.24 \text{ (deg)}}$$